REASONING UNDER UNDER UNCERTAINTY WITH SUBJECTIVE LOGIC

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About me

- Prof. Audun Jøsang, University of Oslo, 2008
- Research interests
  - Information Security
  - Bayesian Reasoning
- Bio
  - MSc Telecom, NTH, 1988
  - Engineer, Alcatel Telecom
  - MSc Security, London 1993
  - PhD Security, NTNU 1998
  - A.Prof. QUT, Australia, 2000

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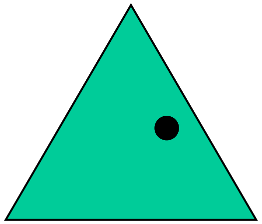
Tutorial overview

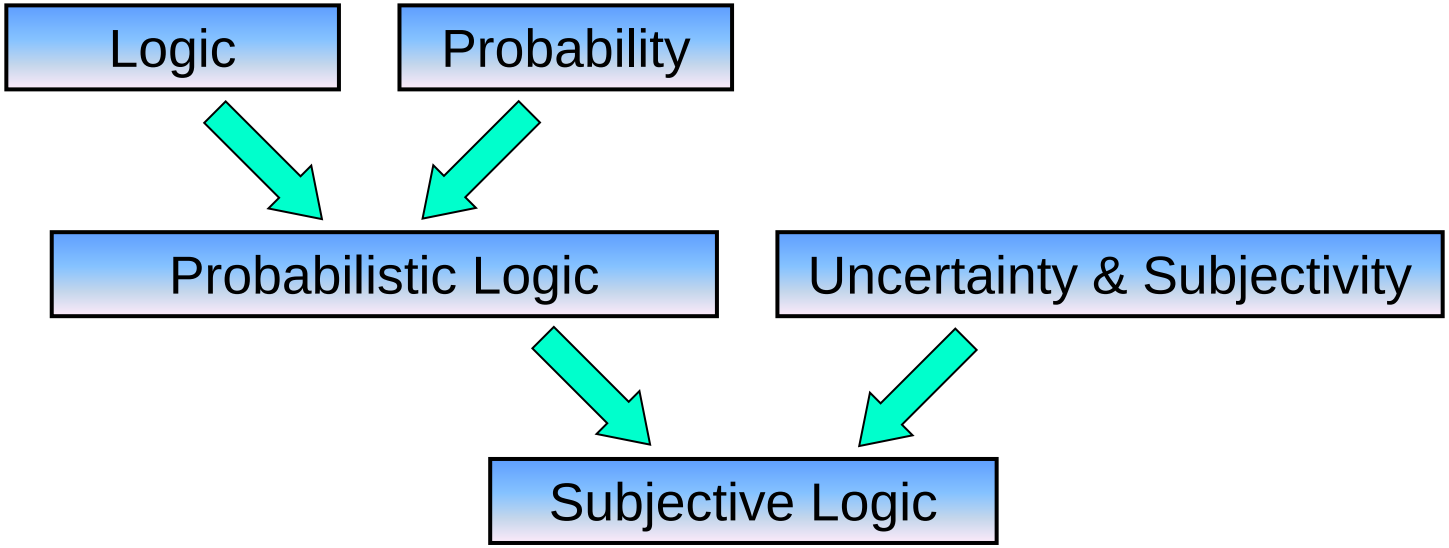
1. Representations of subjective opinions



2. Operators of subjective logic

3. Applications of subjective logic:
   - Trust fusion and transitivity
   - Trust networks
   - Bayesian reasoning
   - Subjective networks

The General Idea of Subjective Logic



Probabilistic Logic Examples

Binary Logic	Probabilistic logic
AND: $x \wedge y$	$p(x \wedge y) = p(x)p(y)$
OR: $x \vee y$	$p(x \vee y) = p(x) + p(y) - p(x)p(y)$
MP: $\{\, x \rightarrow y,\ x \,\} \ \Rightarrow \ y$	$p(y) = p(x)\,p(y\,\vert\,x) + p(\bar{x})\,p(y\,\vert\,\bar{x})$
MT: $\{\, x \rightarrow y, \bar{y} \,\} \ \Rightarrow \ \bar{x}$	$p(x\,\vert\,y) = \dfrac{a(x)\,p(y\,\vert\,x)}{a(x)\,p(y\,\vert\,x) + a(\bar{x})\,p(y\,\vert\,\bar{x})}$ $p(x\,\vert\,\bar{y}) = \dfrac{a(x)\,p(\bar{y}\,\vert\,x)}{a(x)\,p(\bar{y}\,\vert\,x) + a(\bar{x})\,p(\bar{y}\,\vert\,\bar{x})}$ $p(x) = p(y)\,p(x\,\vert\,y) + p(\bar{y})\,p(x\,\vert\,\bar{y})$

a: base rate

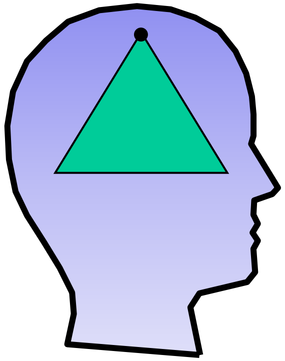
Probability and Uncertainty

<u>Frequentist (aleatory):</u>

- Confident when based on much observation evidence
- Unconfident when based on little observation evidence
- *E.g.: Probability of heads when flipping coin is ½ and confident*

<u>Subjective (epistemic):</u>

- Confident when dynamics of situation are known
- Unconfident when dynamics of situation are unknown
- *E.g. Probability of Oswald killed Kennedy is ½ but unconfident*

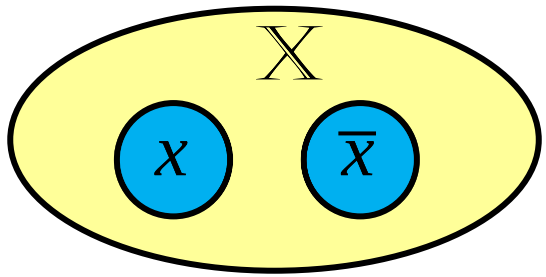


Domains, variables and opinions

Binary domain $\mathbb{X} = \{x, \overline{x}\}$

Binary variable $X = x$

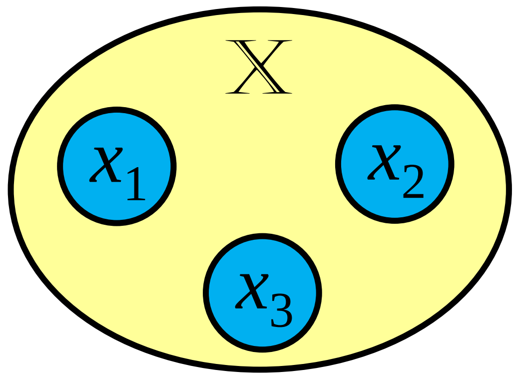
Binomial opinion



3-ary domain $\mathbb{X}$

Random variable $X \in \mathbb{X}$

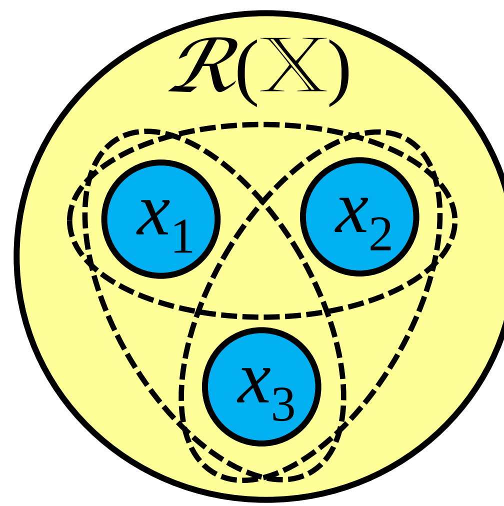
Multinomial opinion



Hyperdomain $\mathcal{R}(\mathbb{X})$

Hypervariable $X \in \mathcal{R}(\mathbb{X})$

Hypernomial opinion



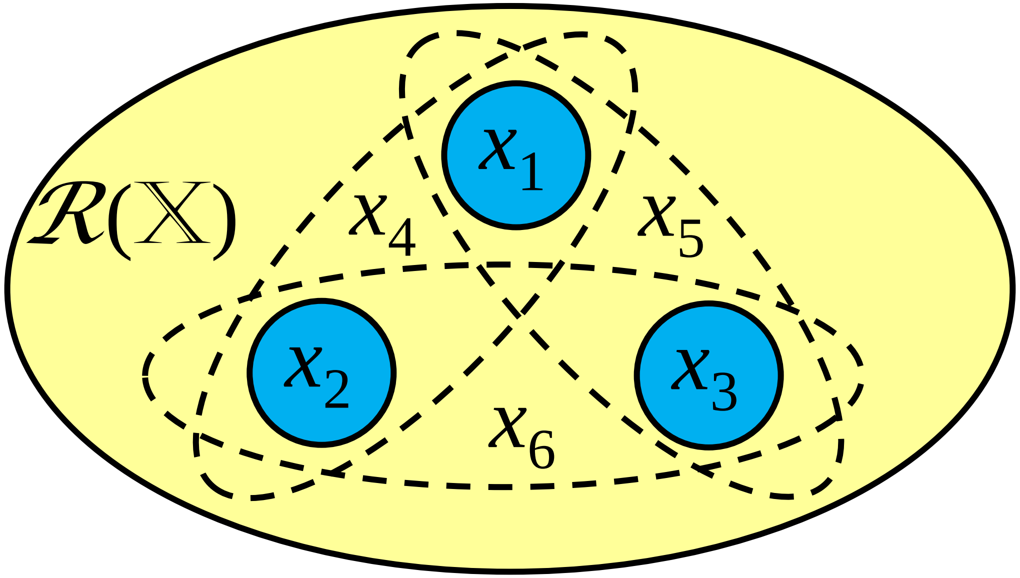
Domains and Hyperdomains

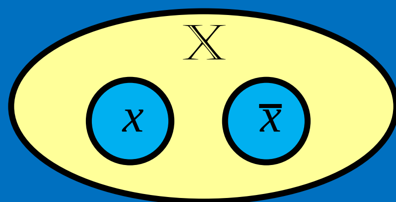
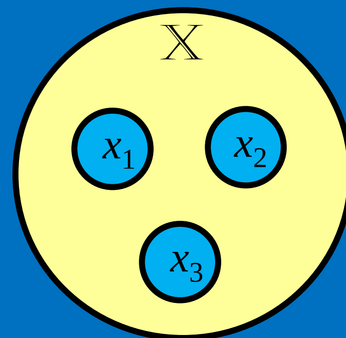
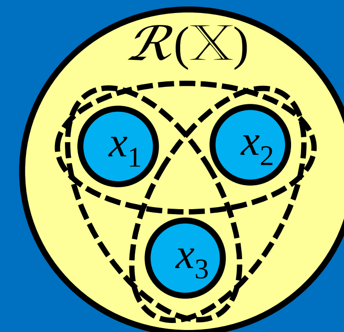
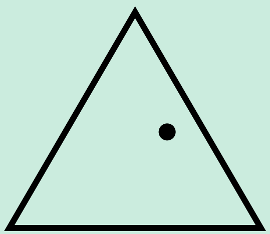
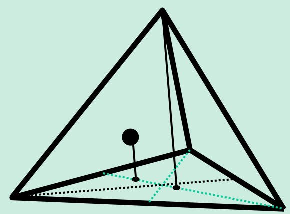
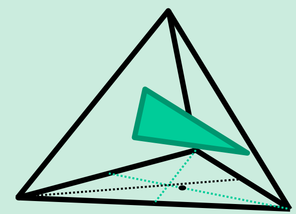
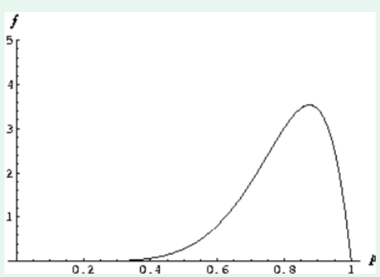
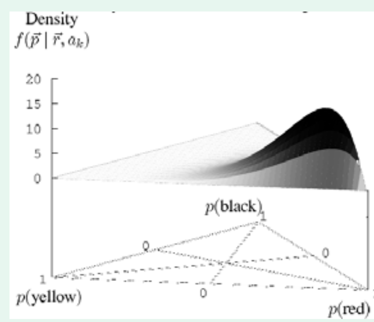
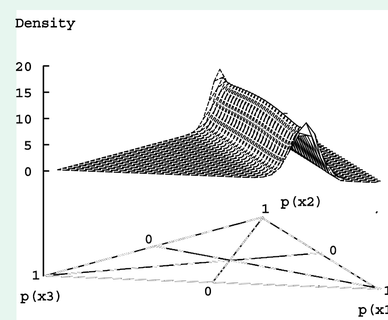
- A domain $\mathbb{X}$ is a state space of distinct possibilities
- Powerset $\mathcal{P}(\mathbb{X}) = 2^{\mathbb{X}}$, set of subsets, including $\{\mathbb{X}, \emptyset\}$
- Reduced powerset $\mathcal{R}(\mathbb{X}) = \mathcal{P}(\mathbb{X}) \setminus \{\mathbb{X}, \emptyset\}$
- $\mathcal{R}(\mathbb{X}) = \{ x_1, x_2, x_3, x_4, x_5, x_6 \}$
- $\mathcal{R}(\mathbb{X})$ called Hyperdomain
- Cardinalities

  $|\mathbb{X}| = 3$ in this example
- $|\mathcal{P}(\mathbb{X})| = 2^{|\mathbb{X}|}$

  $= 8$ in this example
- $|\mathcal{R}(\mathbb{X})| = 2^{|\mathbb{X}|} - 2$

  $= 6$ in this example



	Binomial Opinion **Binary domain $\mathbb{X}$** **Binary variable $X = x$**	**Multinomial Opinion** **n-ary domain $\mathbb{X}$** **Random variable $X \in \mathbb{X}$**	**Hypernomial Opinion** **hyperdomain $\mathcal{R}(\mathbb{X})$** **Hypervariable $X \in \mathcal{R}(\mathbb{X})$**
Domain			
Opinion represen-tation			
PDF represen-tation	Beta PDF over x  FIG 1: Beta function after 7 positive and 1 negative results	Dirichlet PDF over $\mathbb{X}$ 	Hyper Dirichlet over $\mathbb{X}$ 

Binomial subjective opinions

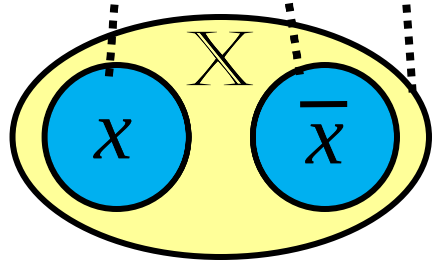
- Belief mass and base rate on binary domains
  - $b_x^A = b(x)$ is observer A's belief in x
  - $d_x^A = b(\overline{x})$ is observer A's disbelief in x
  - $u_x^A = b(X)$ is observer A's uncertainty about x
  - a_x^A is the base rate of x

Binomial opinion

$$\omega_x^A = (b_x^A, d_x^A, u_x^A, a_x^A)$$

base rate of x

Binary domain



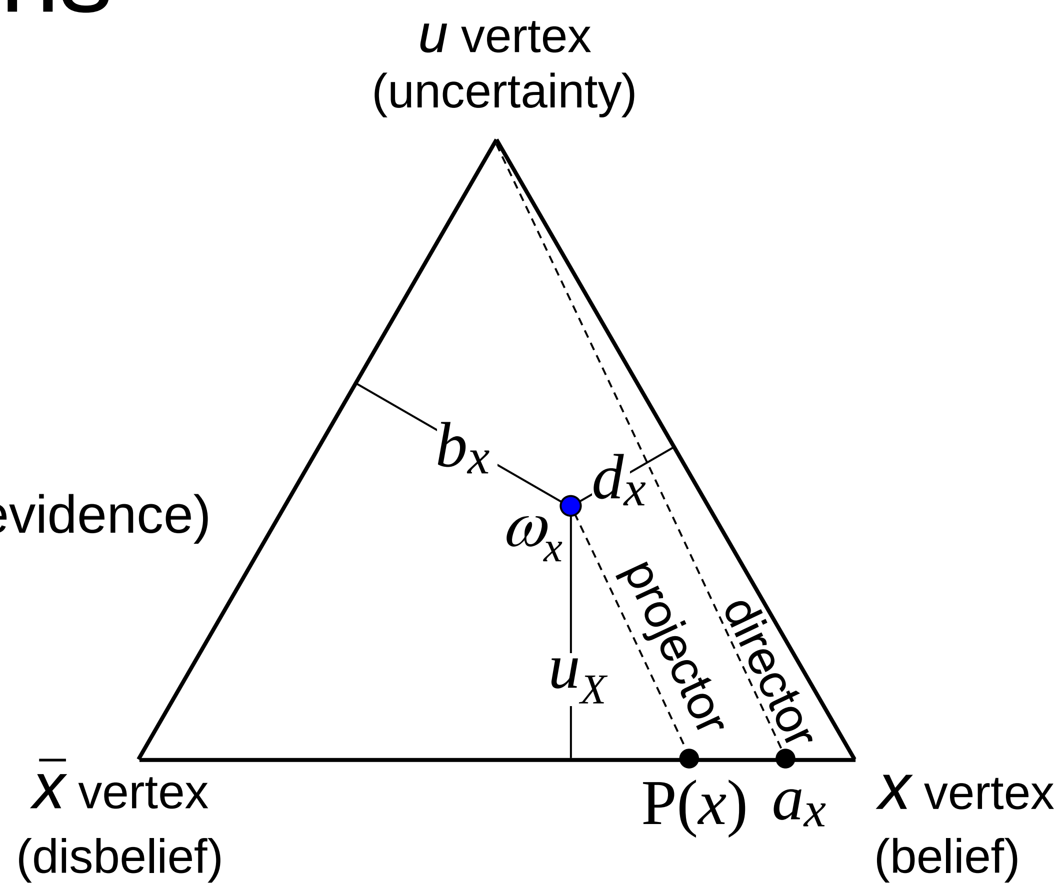
$$b_x^A + d_x^A + u_x^A = 1$$

Binomial opinions

- Ordered quadruple:

$$\omega_x = (b_x, d_x, u_x, a_x)$$

  - b_x : belief
  - d_x : disbelief
  - u_x : uncertainty (vacuity of evidence)
  - a_x : base rate

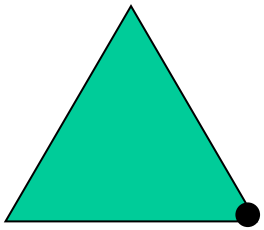


- $b_x + d_x + u_x = 1$

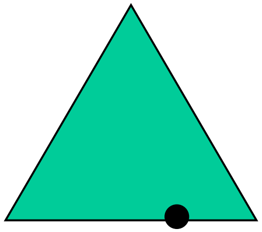
- Projected probability: $\mathrm{P}(x) = b_x + a_x \cdot u_x$

Example ω_x =(0.4, 0.2, 0.4, 0.9), $\mathrm{P}(x) = 0.76$

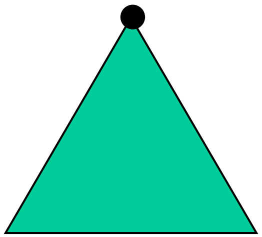
Opinion types



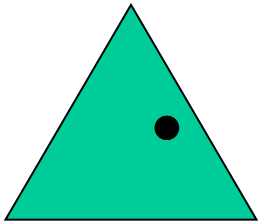
Absolute opinion: b_x=1 .
Equivalent to TRUE.



Dogmatic opinion: u_x=0 .
Equivalent to probabilities.



Vacuous opinion: u_x=1 .
Equivalent to UNDEFINED.



General uncertain opinion: $u_x \neq 0$.

Beta PDF representation

$$\text{Beta}\,(p(x), \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, p(x)^{\alpha-1}(1-p(x))^{\beta-1}$$

$$\alpha = r + Wa$$

$$\beta = s + W(1\text{-}a)$$

r: # observations of x

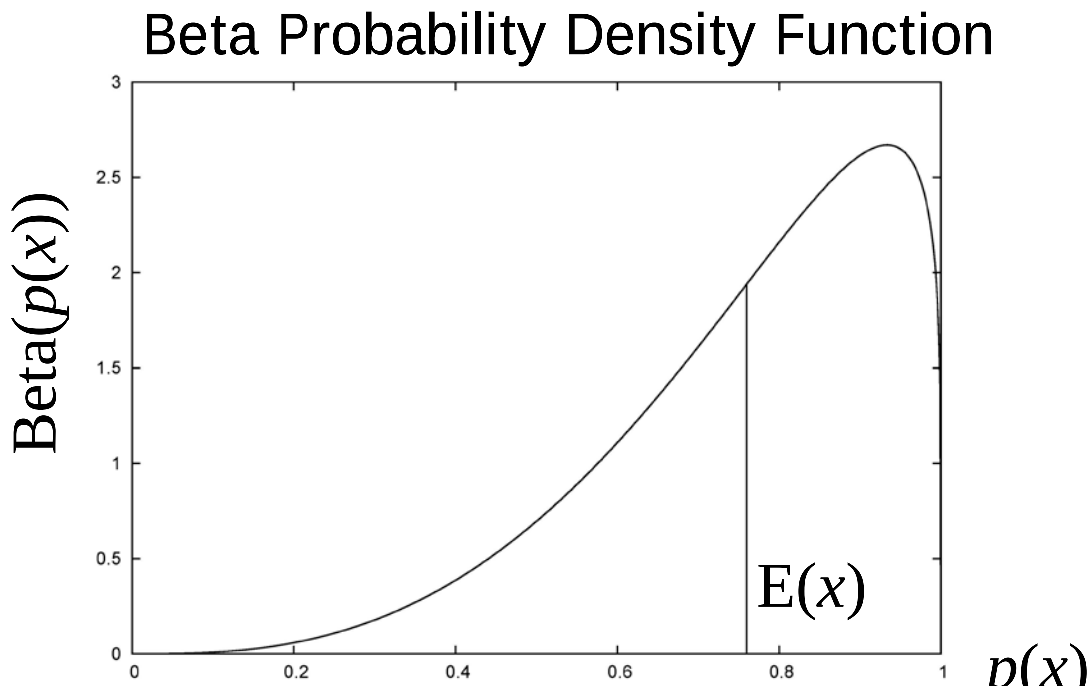
s: # observations of $\overline{x}$

a: base rate of x

$W = 2$: non-informative prior weight

$\mathrm{E}(x)$: Expected probability

$\mathrm{E}(x) = \mathrm{P}(x)$



Example: $r = 2$, $\mathrm{s} = 1$, $a = 0.9$, $\mathrm{E}(x) = 0.76$

Binomial Opinion $\leftrightarrow$ Beta PDF

- (r,s,a) represents Beta PDF evidence parameters.
- (b,d,u,a) represents binomial opinion.
- $\mathrm{P}(x) = \mathrm{E}(x)$
- Op $\rightarrow$ Beta: $\begin{cases} r = Wb/u \\ s = Wd/u \\ b+d+u=1 \end{cases}$

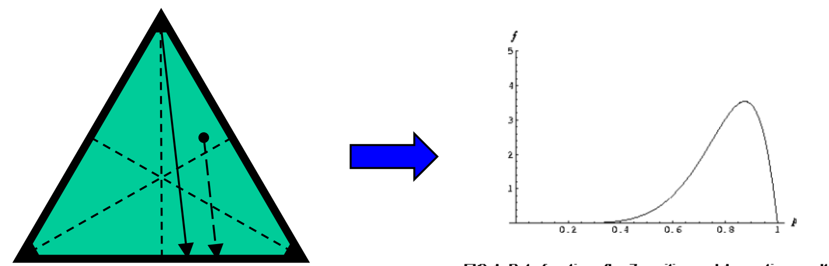


FIG 1: Beta function after 7 positive and 1 negative results

- Beta $\rightarrow$ Op: $\begin{cases} b = \frac{r}{r+s+W} \\ d = \frac{s}{r+s+W} \\ u = \frac{W}{r+s+W} \end{cases}$

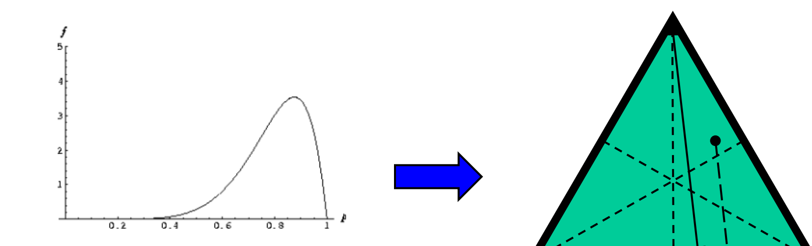
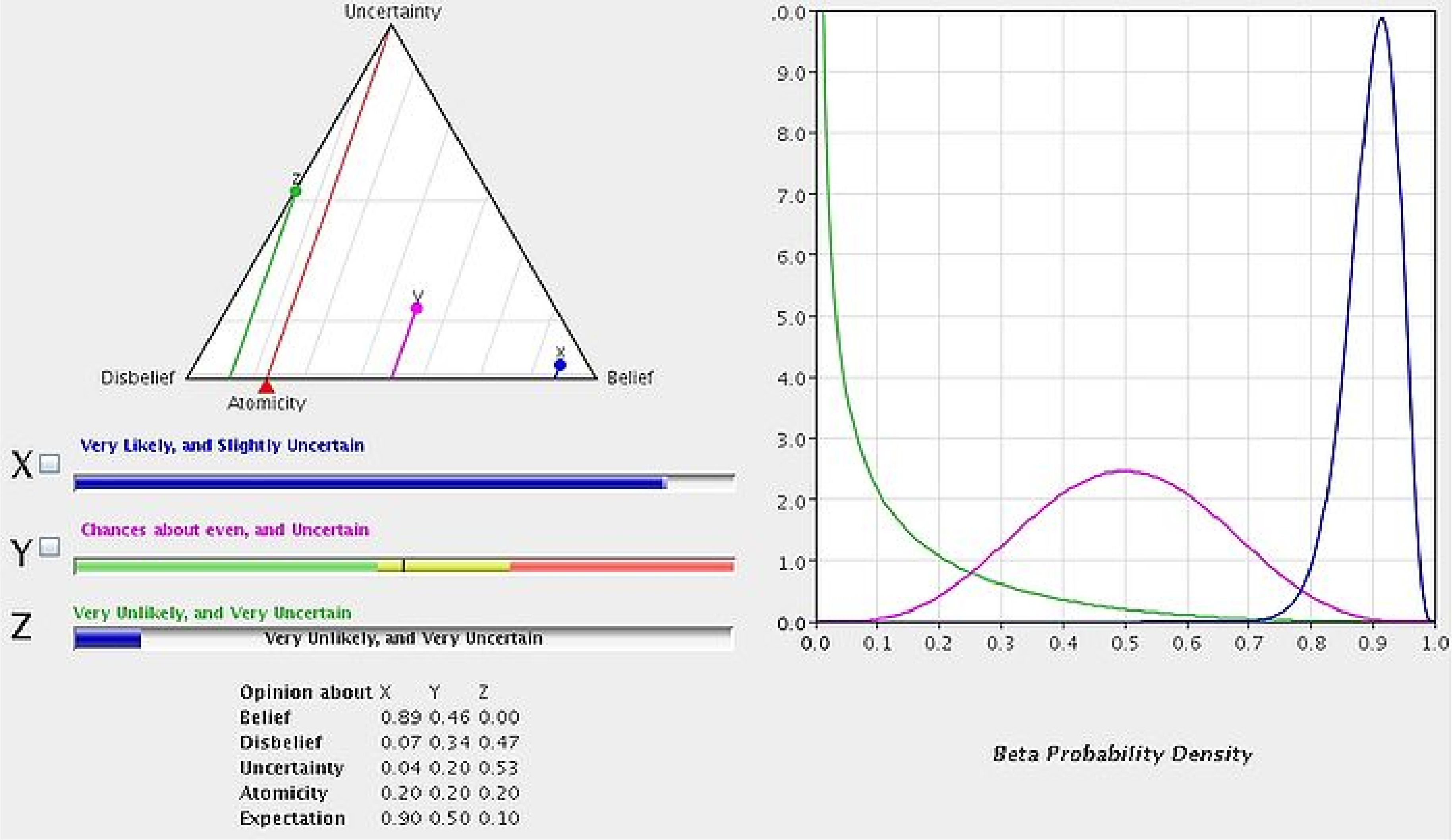


FIG 1: Beta function after 7 positive and 1 negative results

$W = 2$

Online demo



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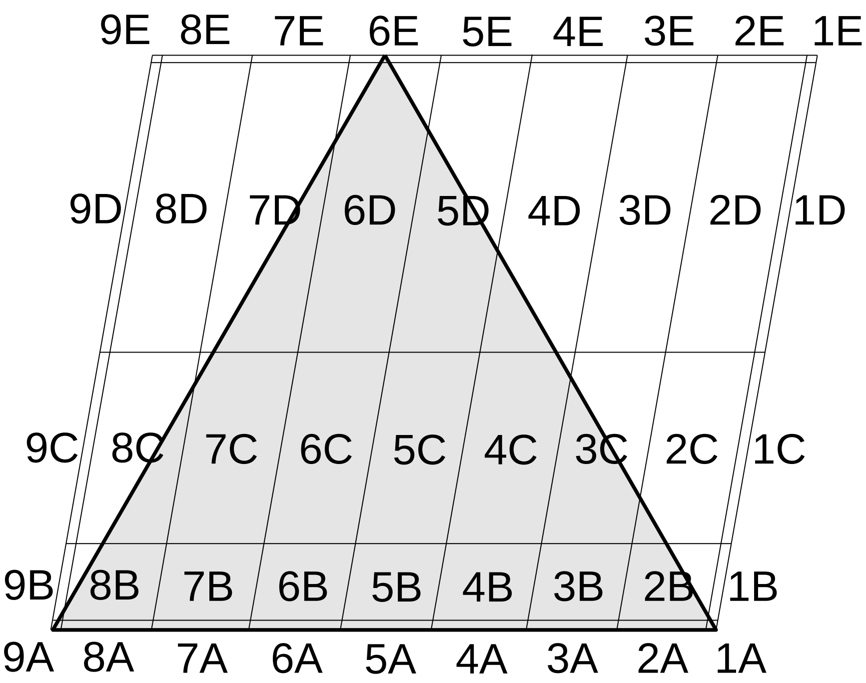
Likelihood and Confidence

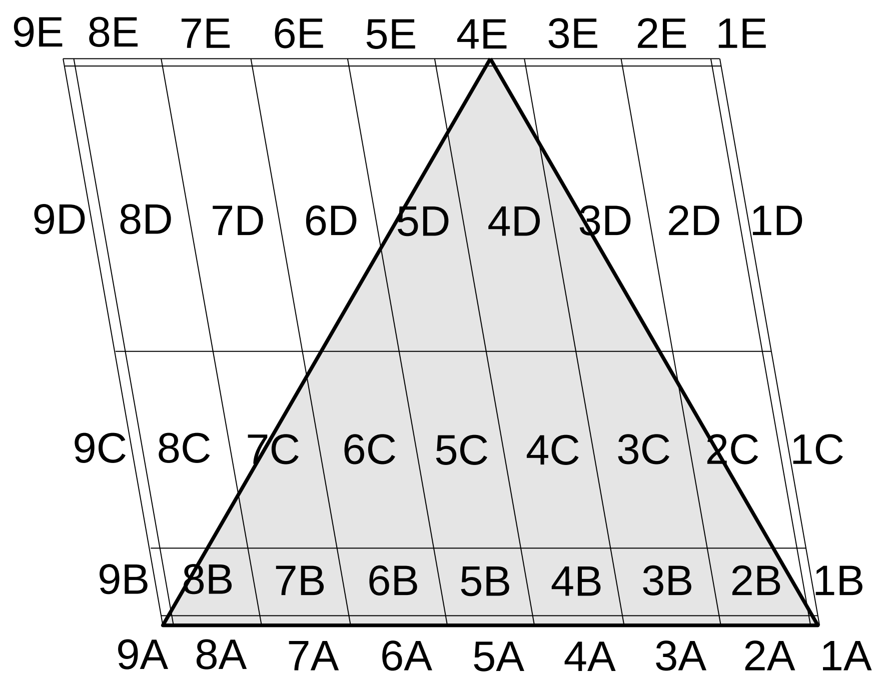
Likelihood categories:		Absolutely not	Very unlikely	Unlikely	Somewhat unlikely	Chances about even	Somewhat likely	Likely	Very likely	Absolutely
Confidence categories:		**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
No confidence	**E**	9E	8E	7E	6E	5E	4E	3E	2E	1E
Low confidence	**D**	9D	8D	7D	6D	5D	4D	3D	2D	1D
Some confidence	**C**	9C	8C	7C	6C	5C	4C	3C	2C	1C
High confidence	**B**	9B	8B	7B	6B	5B	4B	3B	2B	1B
Total confidence	**A**	9A	8A	7A	6A	5A	4A	3A	2A	1A

Mapping qualitative to opinion

- Category mapped to corresponding field of triangle
- Mapping depends on base rate
- Non-existent categories depending on base-rates



base rate $a = 1/3$



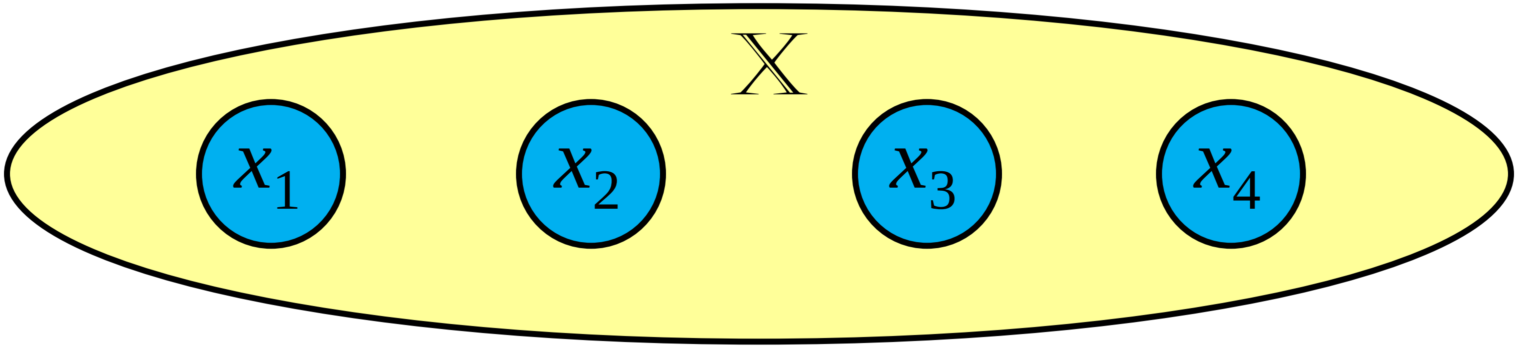
base rate $a = 2/3$

Mapping categories to opinions

- Overlay category matrix with opinion triangle
- Matrix skewed as a function of base rate
- Not all categories map to opinions
  - For a low base rate, it is impossible to describe an event as highly likely and uncertain, but possible to describe it as highly unlikely and uncertain.
  - E.g. with regard to tuberculosis which has a low base rate, it would be wrong to say that a patient is likely to be infected, with high uncertainty. Similarly it would be possible to say that the patient is probably not infected, with high uncertainty

Multinomial domain

- Generalisation of binary domain
- Set of exclusive and exhaustive singletons.
- Example domain: $\mathbb{X}=\{x_1, x_2, x_3, x_4\}$, $|\mathbb{X}| = 4$.



Multinomial Opinions

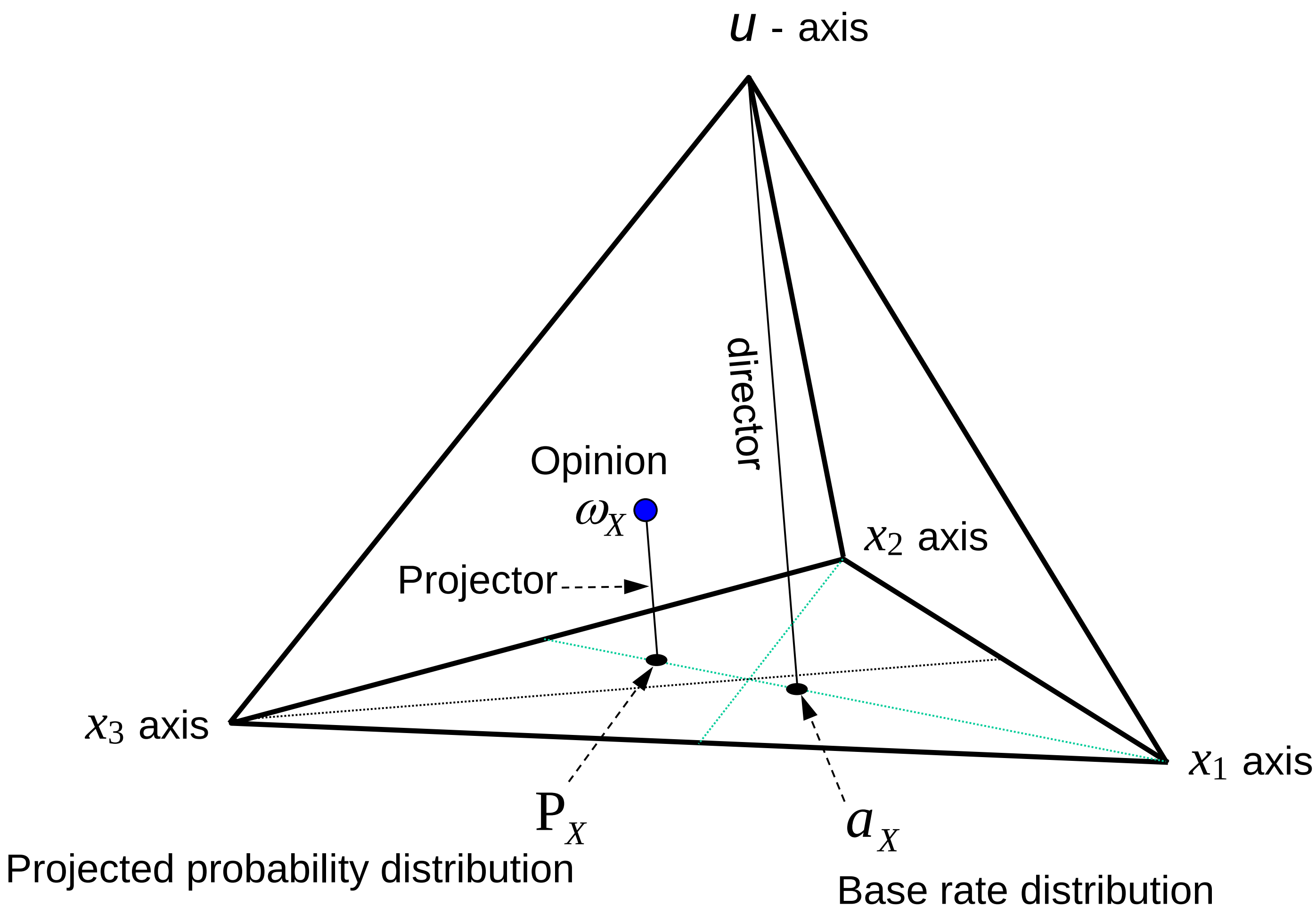
- Domain: $\mathbb{X}=\{x_1 \ldots x_k\}$
- Random variable $X \in \mathbb{X}$
- Multinomial opinion: $\omega_X = (b_X, u_X, a_X)$

- Belief mass distribution b_X where $u + \Sigma b_X(x) = 1$

  $b_X(x)$ is belief mass on $x \in \mathbb{X}$
- Uncertainty mass: u_X is a single value in range $[0,1]$
- Base rate distribution a_X where $\Sigma a_X(x) = 1$

  $a_X(x)$ is base rate of $x \in \mathbb{X}$
- Projected probability: $\mathrm{P}_X(x) = b_X(x) + a_X(x) \cdot u_X$

Opinion tetrahedron (ternary domain)



Dirichlet PDF representation

$$\mathrm{Dir}(p_X) = \frac{\Gamma\left(\sum_{i=1}^{k} \alpha_X(x_i)\right)}{\prod_{i=1}^{k} \Gamma(\alpha_X(x_i))} \prod_{i=1}^{k} p_X(x_i)^{\alpha(x_i)-1}$$

$$\Sigma\, p_X(x_i) = 1$$

$$\alpha_X(x_i) = r_X(x_i) + W \cdot a_X(x_i)$$

$r_X(x_i)$: # observations of x_i

$a_X(x_i)$: base rate of x_i

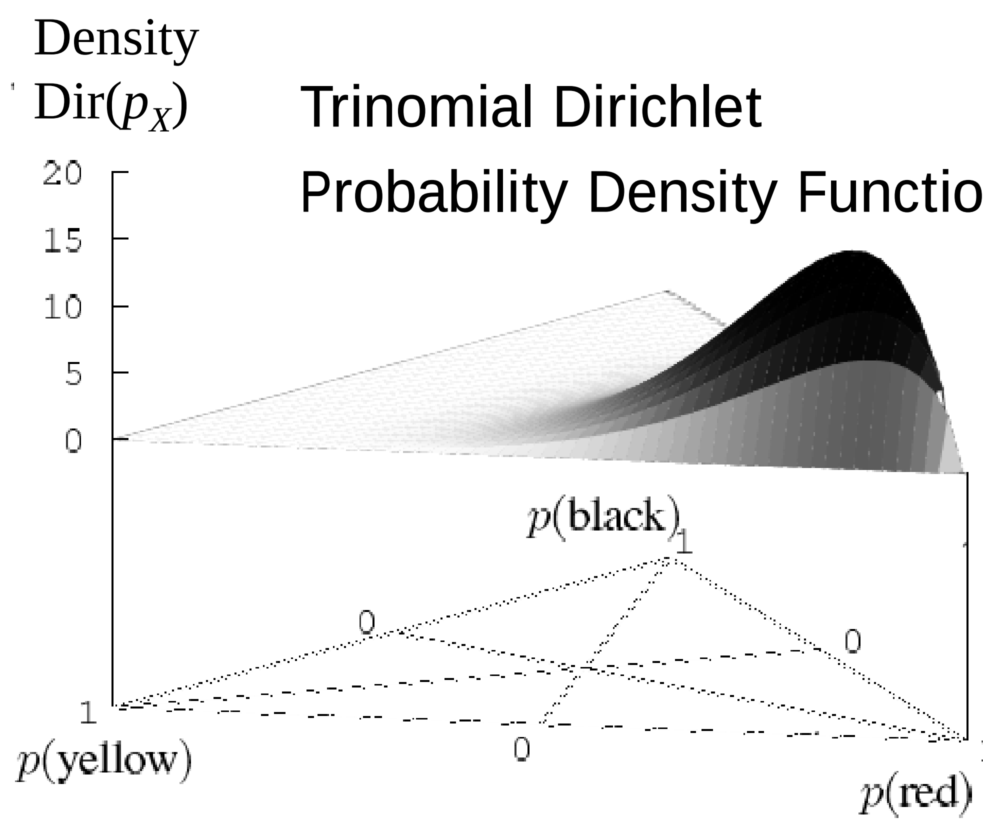
E_X: Expected proba. distr.

$E_X = P_X$

Example:

- 6 red balls
- 1 yellow ball
- 1 black ball

Trinomial Dirichlet Probability Density Function



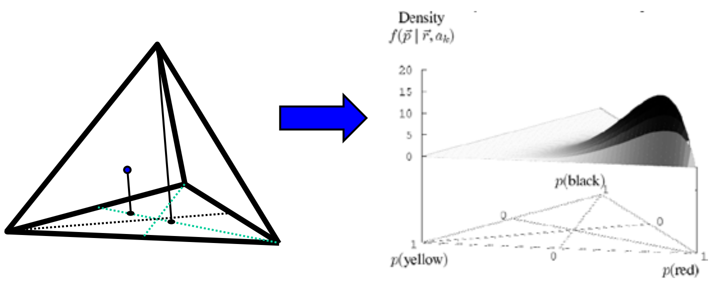
Multinomial Opinion ↔ Dirichlet PDF

- Dirichlet PDF evidence parameters: (r_X, a_X)

- Multinomial opinion parameters: (b_X, u_X, a_X)

- Op → Dir:

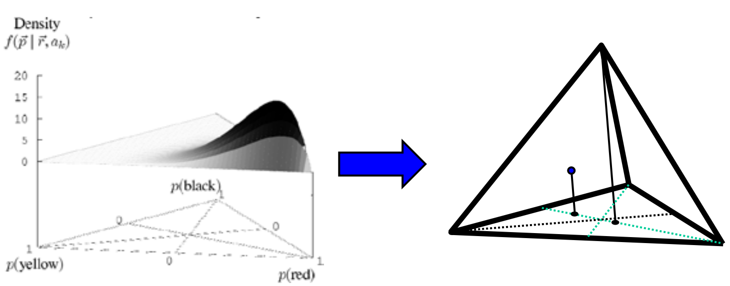
$$\begin{cases} r_X(x) = \dfrac{W \cdot b_X(x)}{u_X} \\ \\ u_X + \sum b_X(x) = 1 \end{cases}$$



$W = 2$

- Dir → Op:

$$\begin{cases} b_X(x) = \dfrac{r_X(x)}{W + \sum r_X(x)} \\ \\ u_X = \dfrac{W}{W + \sum r_X(x)} \end{cases}$$



Non-informative prior weight: W

- Value normally set to $W = 2$.
- When W is equal to the frame cardinality, then the prior Dirichlet PDF is a uniform.
- Normally required that the prior Beta is uniform, which dictates $W = 2$
- Beta PDF is a binomial Dirichlet PDF
- Setting $W > 2$ would make Dirichlet PDF insensitive to new observations, which would be an inadequate model.

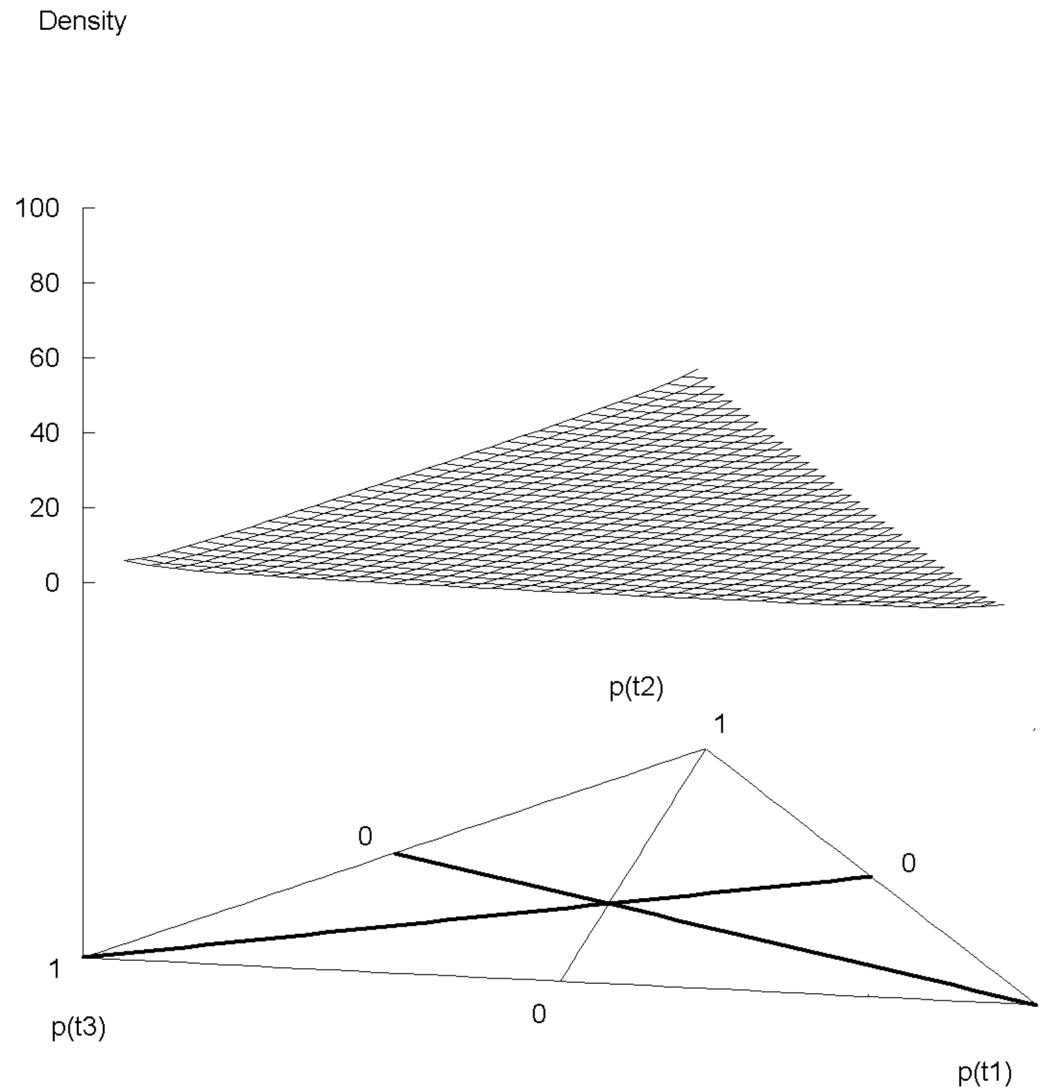
Prior trinomial Dirichlet PDF, $W = 2$

Example:

Urn with balls of 3 different colours.

- t1: Red
- t2: Yellow
- t3: Black

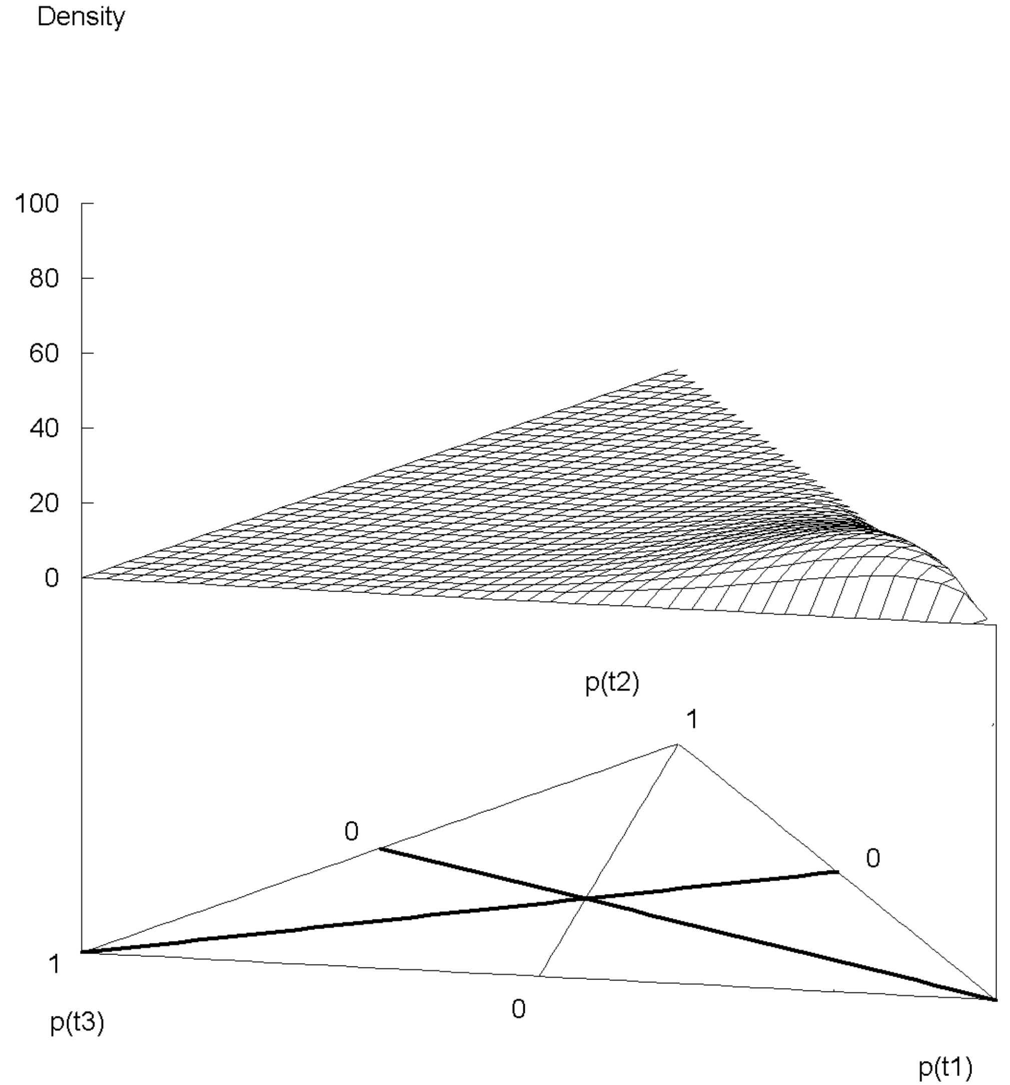
Ternary *a priori* probability density.



Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

- 6 red balls (t1)
- 1 yellow ball (t2)
- 1 black ball (t3)
- $W = 2$

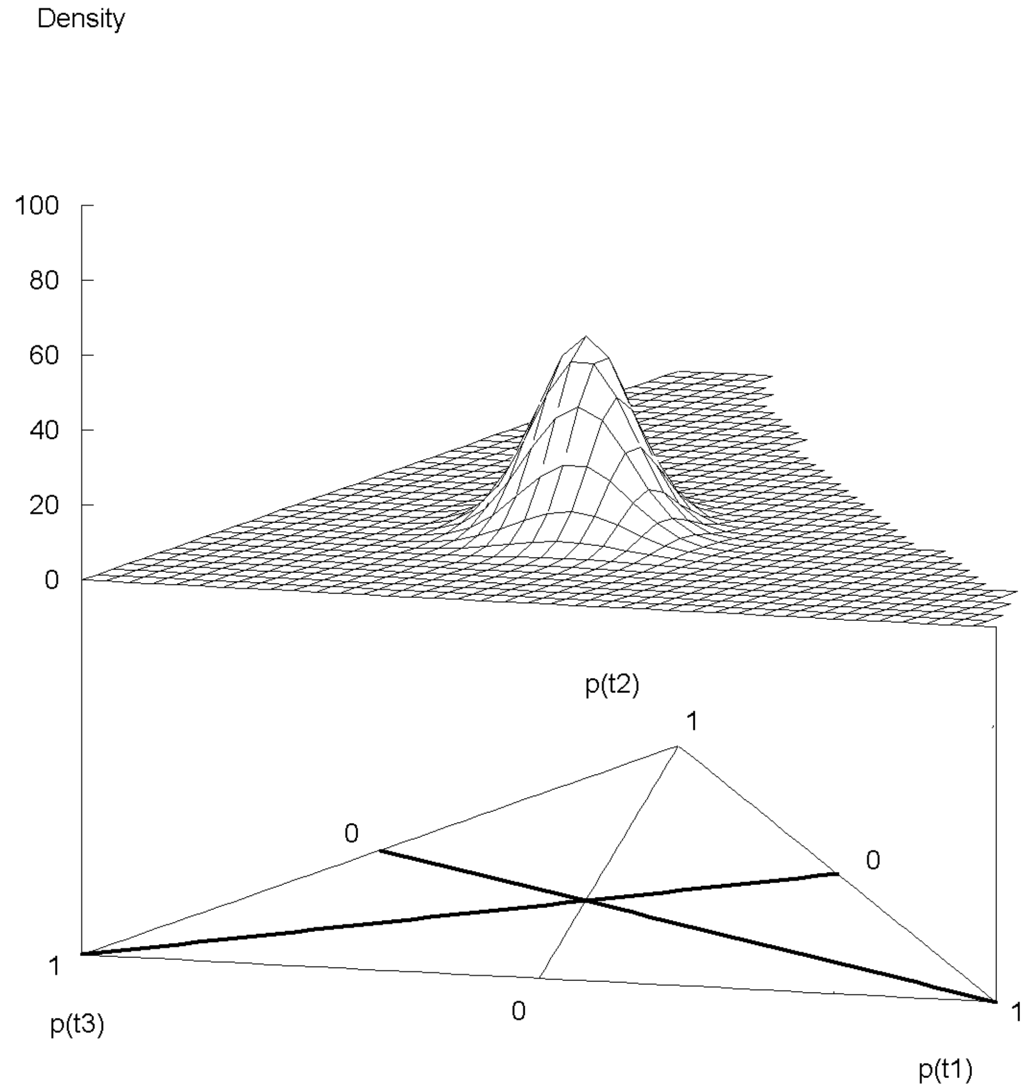


Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

- 20 red balls (t1)
- 20 yellow balls (t2)
- 20 black balls (t3)

- $W = 2$

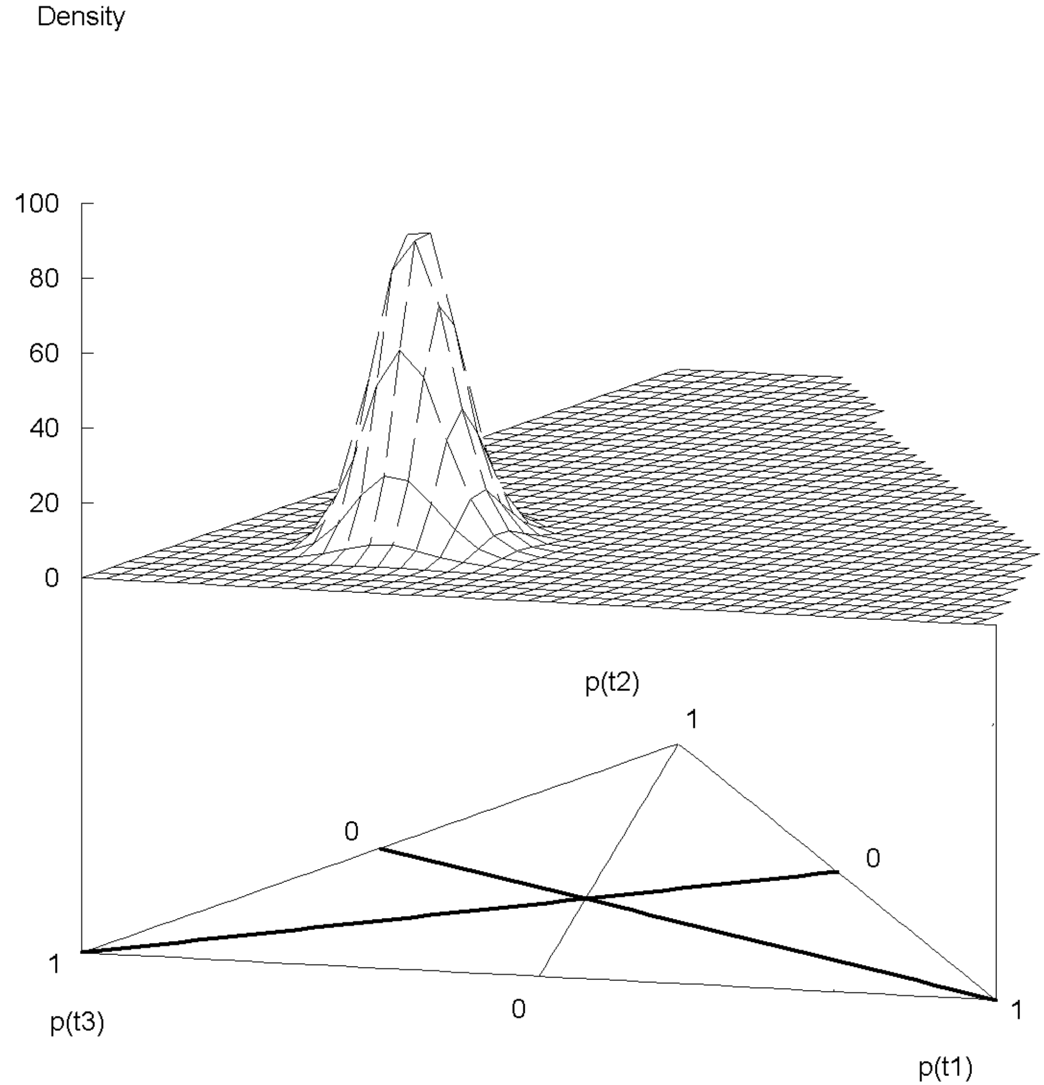


Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

- 20 red balls (t1)
- 20 yellow balls (t2)
- 50 black balls (t3)

- $W = 2$



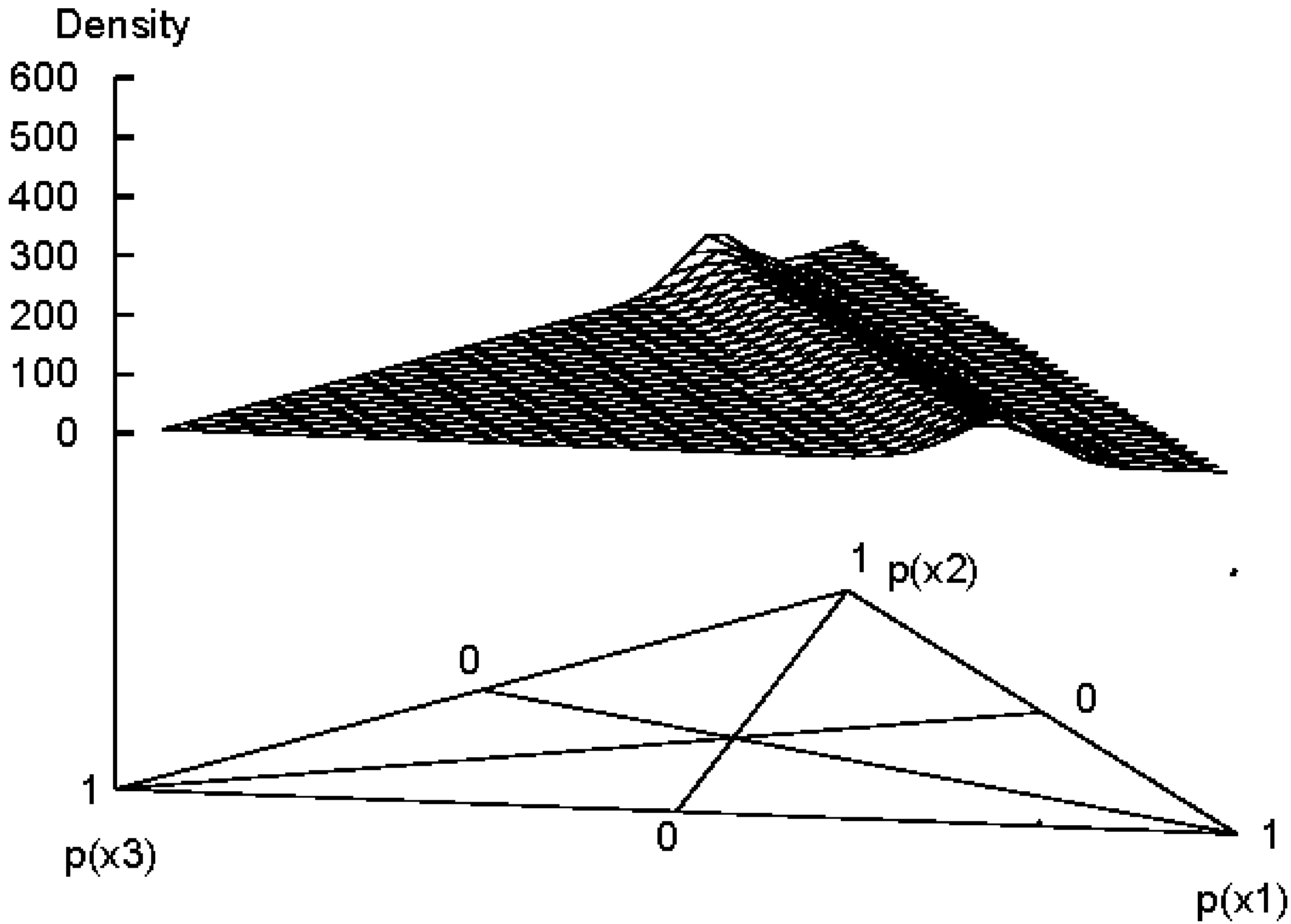
Hyper-Opinions

- Domain: $\mathbb{X} = \{x_1 \ldots x_k\}$
- $\mathcal{P}(\mathbb{X})$ is the powerset of $\mathbb{X}$
- Hyperdomain $\mathcal{R}(\mathbb{X}) = \mathcal{P}(\mathbb{X}) \setminus \{\mathbb{X}, \emptyset\}$
- $\mathcal{R}(\mathbb{X})$ is the reduced powerset
- Hypervariable: $X \in \mathcal{R}(\mathbb{X})$
- Hyper opinion: $\omega_X = (\mathbf{b}_X, u_X, \mathbf{a}_X)$
- Belief mass distribution: $\mathbf{b}_X$ where $u_X + \sum_{X \in \mathcal{R}(\mathbb{X})} \mathbf{b}_X(x) = 1$

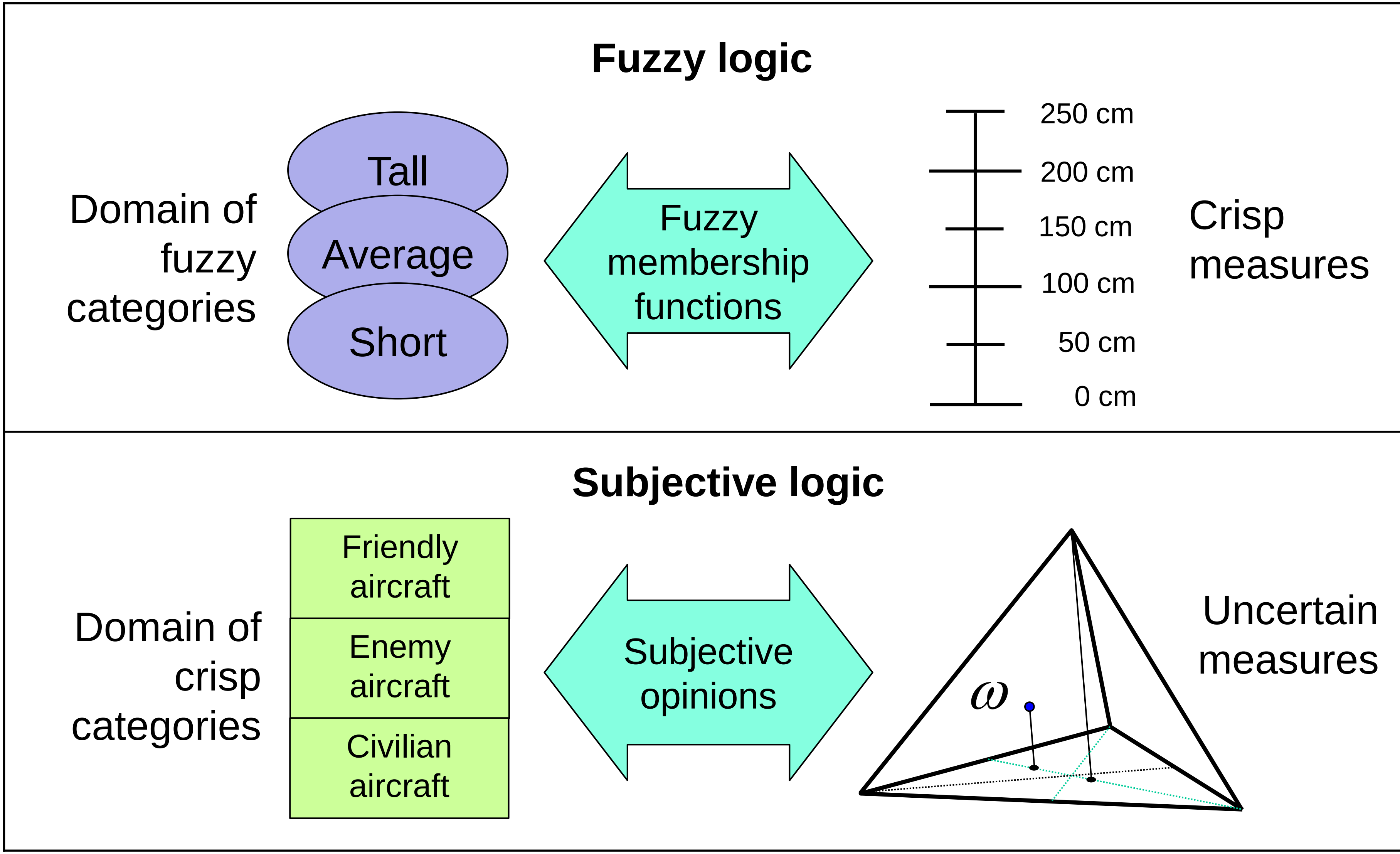
  $\mathbf{b}_X(x)$ is belief mass on $x \in \mathcal{R}(\mathbb{X})$
- Base rate distribution: $\mathbf{a}_X$ where $\sum_{X \in \mathbb{X}} \mathbf{a}_X(x) = 1$

  $\mathbf{a}_X(x)$ is base rate of $x \in \mathbb{X}$
- Proj. probability: $\mathrm{P}_X(x) = \mathbf{a}_X(x) \cdot u_X + \sum_{x_j \in \mathcal{R}(\mathbb{X})} \mathbf{a}_X(x \mid x_j) \cdot \mathbf{b}_X(x_j)$

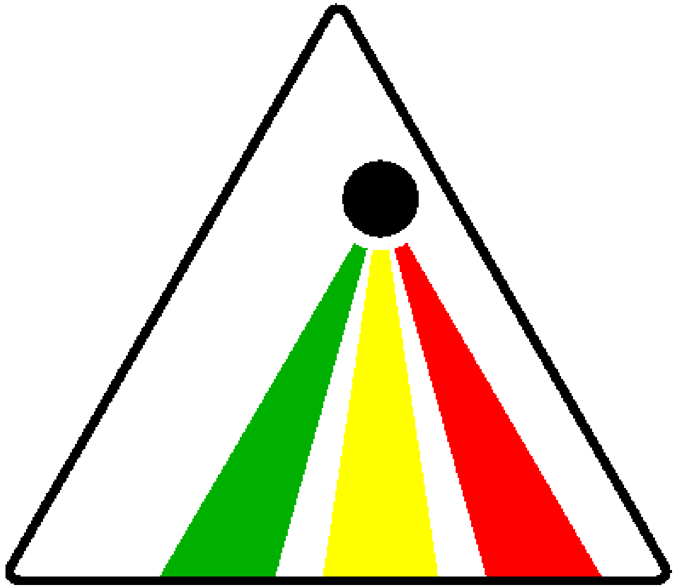
Hyper Dirichlet PDF



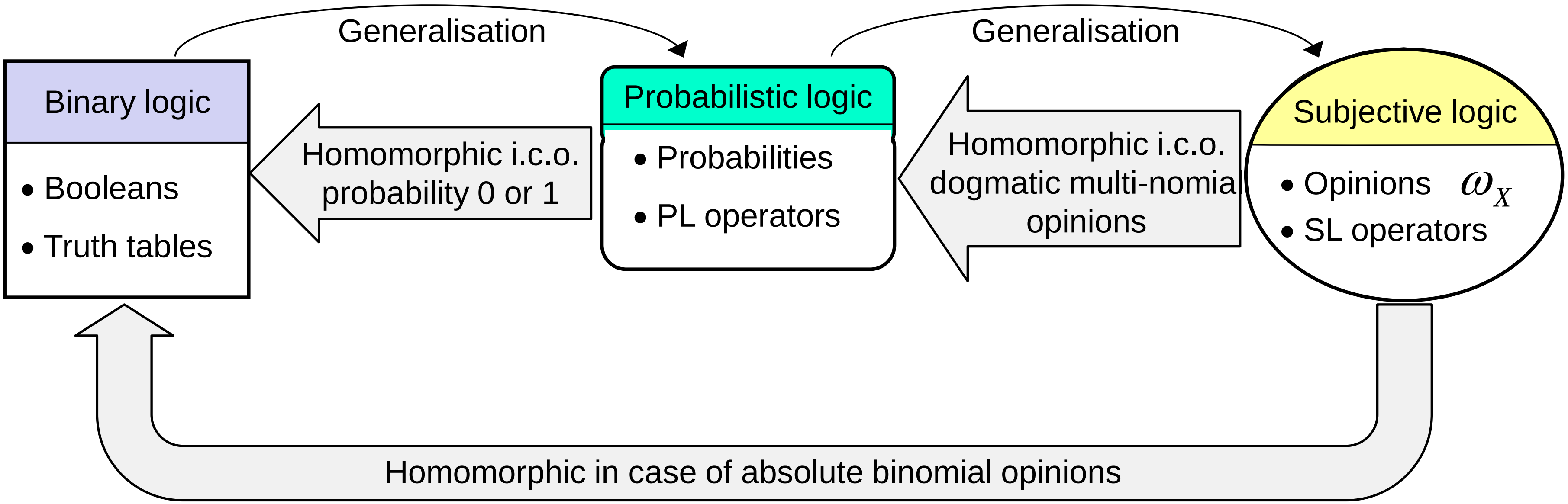
Opinions v. Fuzzy membership functions



Subjective Logic Operators



Homomorphic correspondence



- $P(\omega_x \cdot \omega_y) = P(\omega_x) \cdot P(\omega_y)$ for probabilistic multiplication
- $B(\omega_x \cdot \omega_y) = B(\omega_x) \wedge B(\omega_y)$ for Boolean conjunction

Subjective logic operators 1

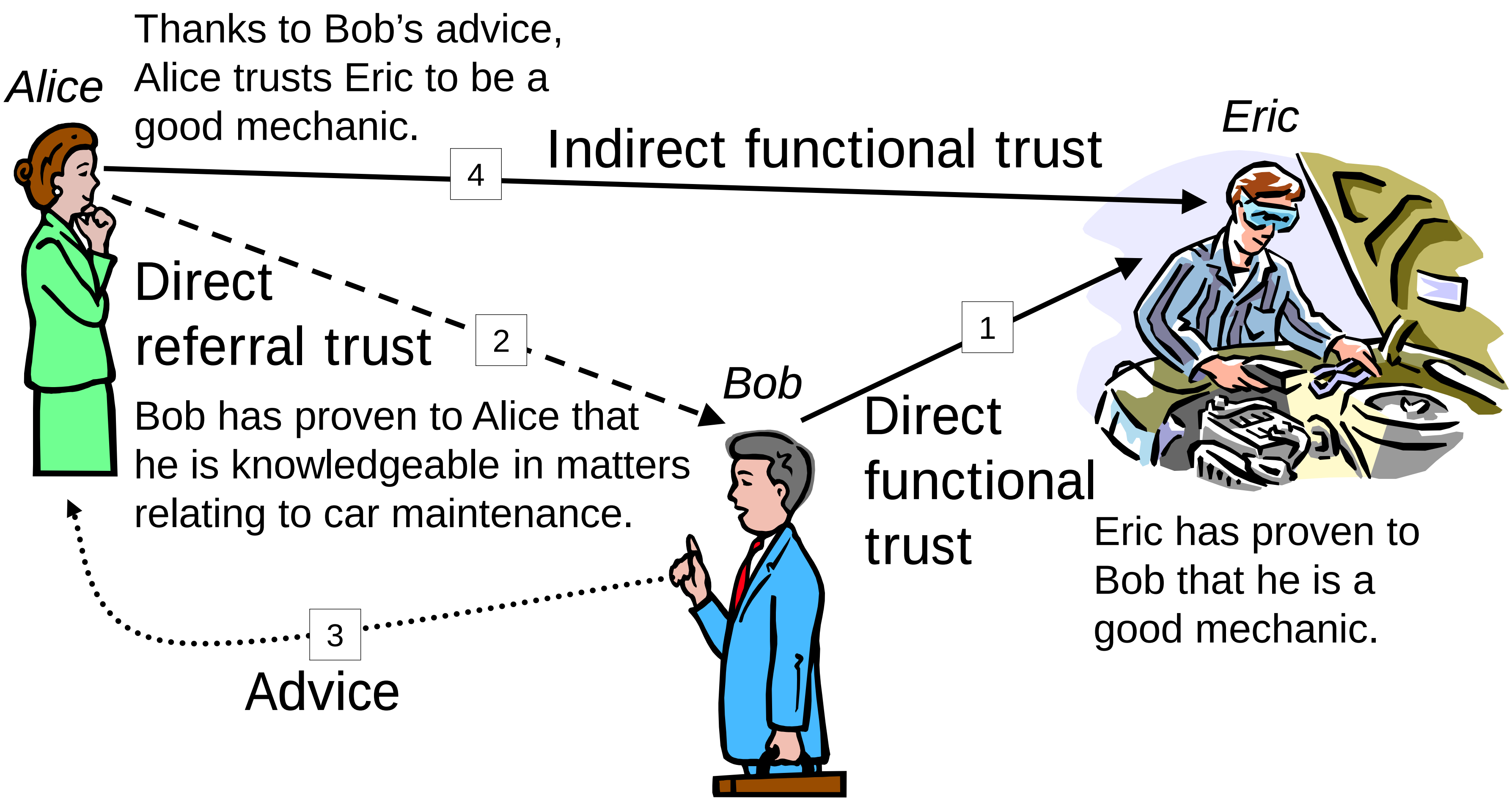
Opinion operator name	Opinion operator symbol	Logic operator symbol	Logic operator name
Addition	+	$\cup$	UNION
Subtraction	-	\	DIFFERENCE
Complement	¬	$\overline{x}$	NOT
Projected probability	P(x)	n.a.	n.a.
Multiplication	·	$\wedge$	AND
Division	/	$\overline{\wedge}$	UN-AND
Comultiplication	$\sqcup$	$\vee$	OR
Codivision	$\overline{\sqcup}$	$\overline{\vee}$	UN-OR

Subjective logic operators 2

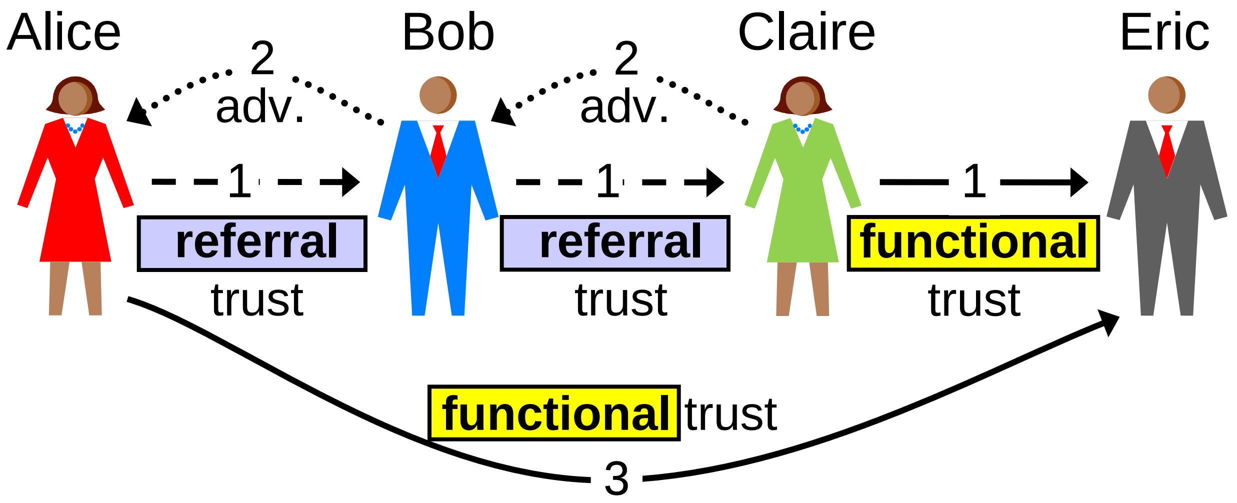
Opinion operator name	**Opinion operator symbol**	**Logic operator symbol**	**Logic operator name**
Transitive discounting	$\otimes$	:	TRANSITIVITY
Cumulative fusion	$\oplus$	$\lozenge$	n.a.
Averaging fusion	$\underline{\oplus}$	$\underline{\lozenge}$	n.a.
Constraint fusion	$\odot$	&	n.a.
Inversion, Bayes' theorem	$\widetilde{\phi}$	$\widetilde{\vert}$	CONTRAPOSITION
Conditional deduction	$\circledcirc$	$\Vert$	DEDUCTION (Modus Ponens)
Conditional abduction	$\widetilde{\circledcirc}$	$\widetilde{\Vert}$	ABDUCTION (Modus Tollens)

Subjective Trust Networks

Trust transitivity



Functional trust derivation requirement

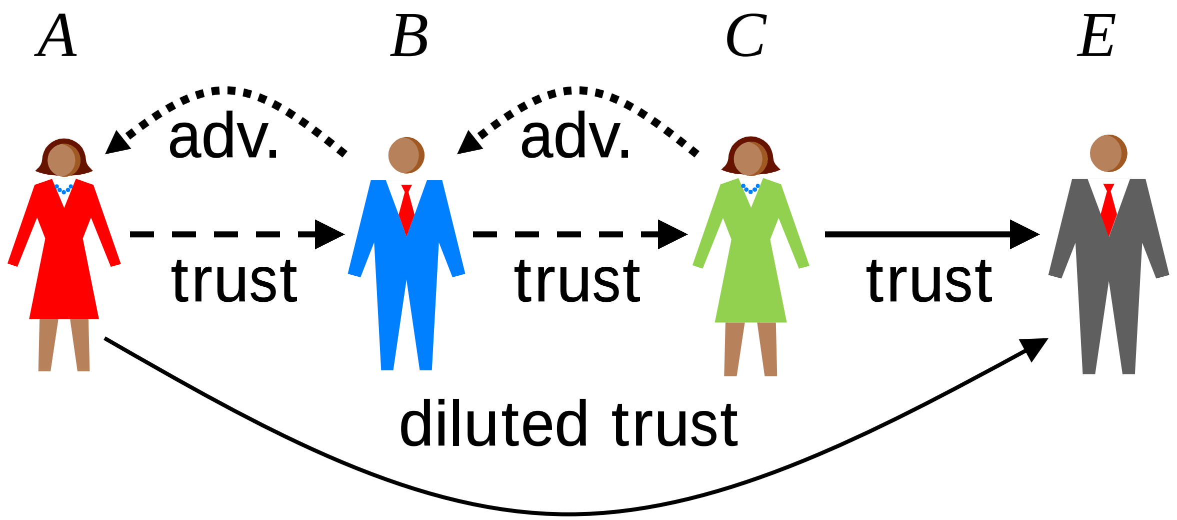


- Functional trust derivation through transitive paths requires that the last trust edge represents functional trust (or an opinion) and that all previous trust edges represent referral trust.

- Functional trust can be an opinion about a variable.

Trust transitivity characteristics

Trust is diluted in a transitive chain.



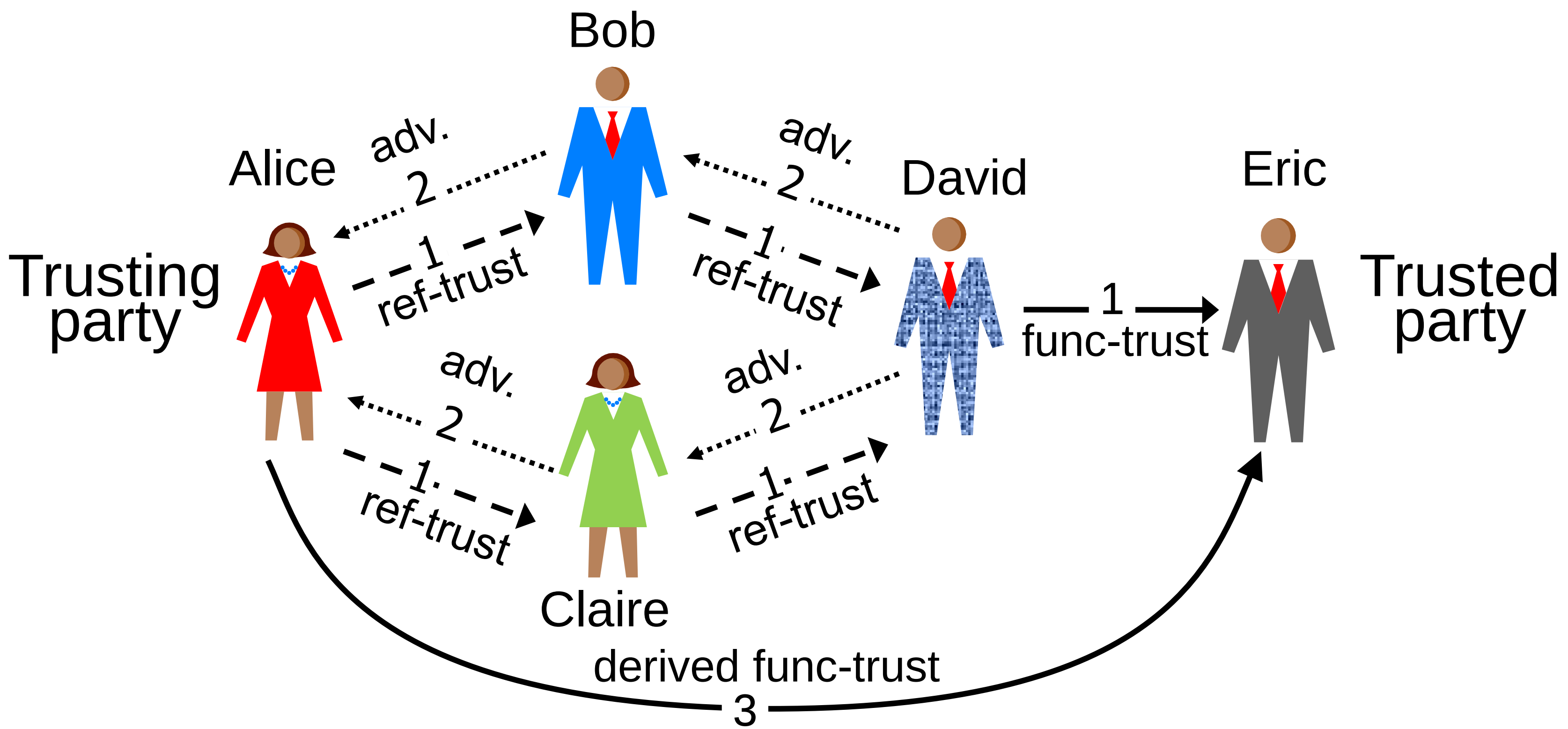
Computed with discounting/transitivity operator of SL

Graph notation: $[A, E] = [A; B] : [B; C] : [C, E]$

SL notation: $\omega_E^{(A;B;C)} = \omega_B^A \otimes \omega_C^B \otimes \omega_E^C$

Trust Fusion

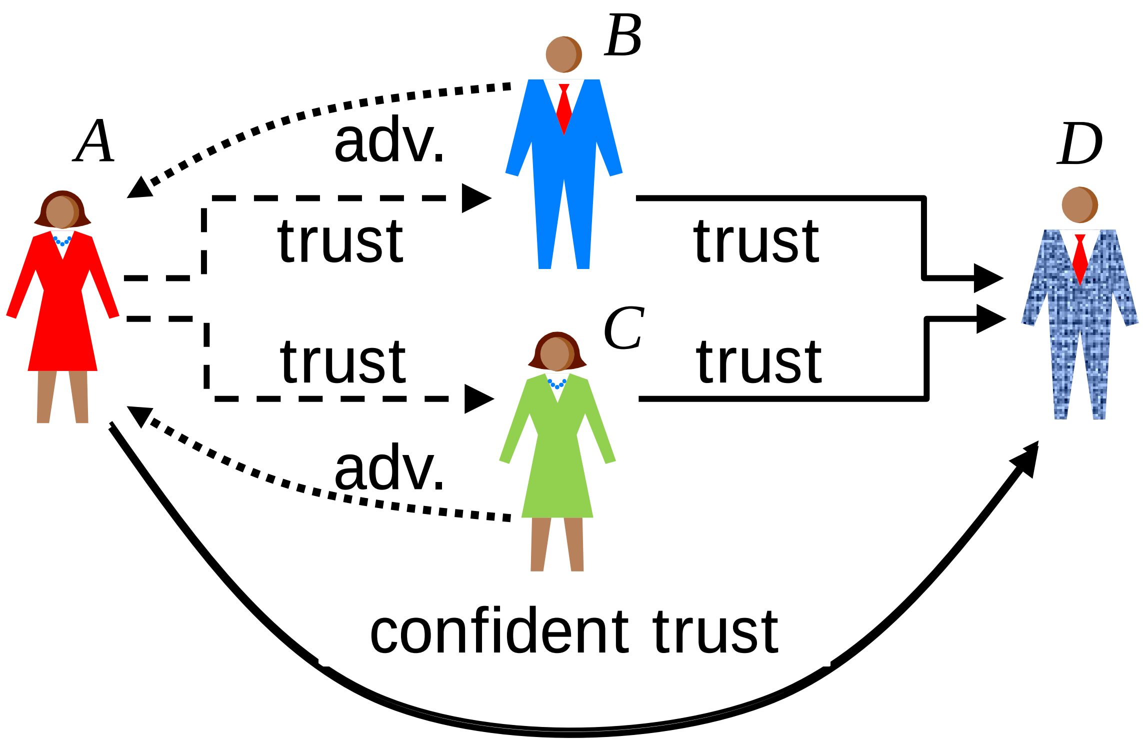
Combination of serial and parallel trust paths



Graph notation: $[A, E] = (([A;B] : [B;D]) \lozenge ([A;C] : [C;D])) : [D,E]$

SL not.: $\omega_E^{[A;B;D]\lozenge[A;C;D]} = ((\omega_B^A \otimes \omega_D^B) \oplus (\omega_C^A \otimes \omega_D^C)) \otimes \omega_E^D$

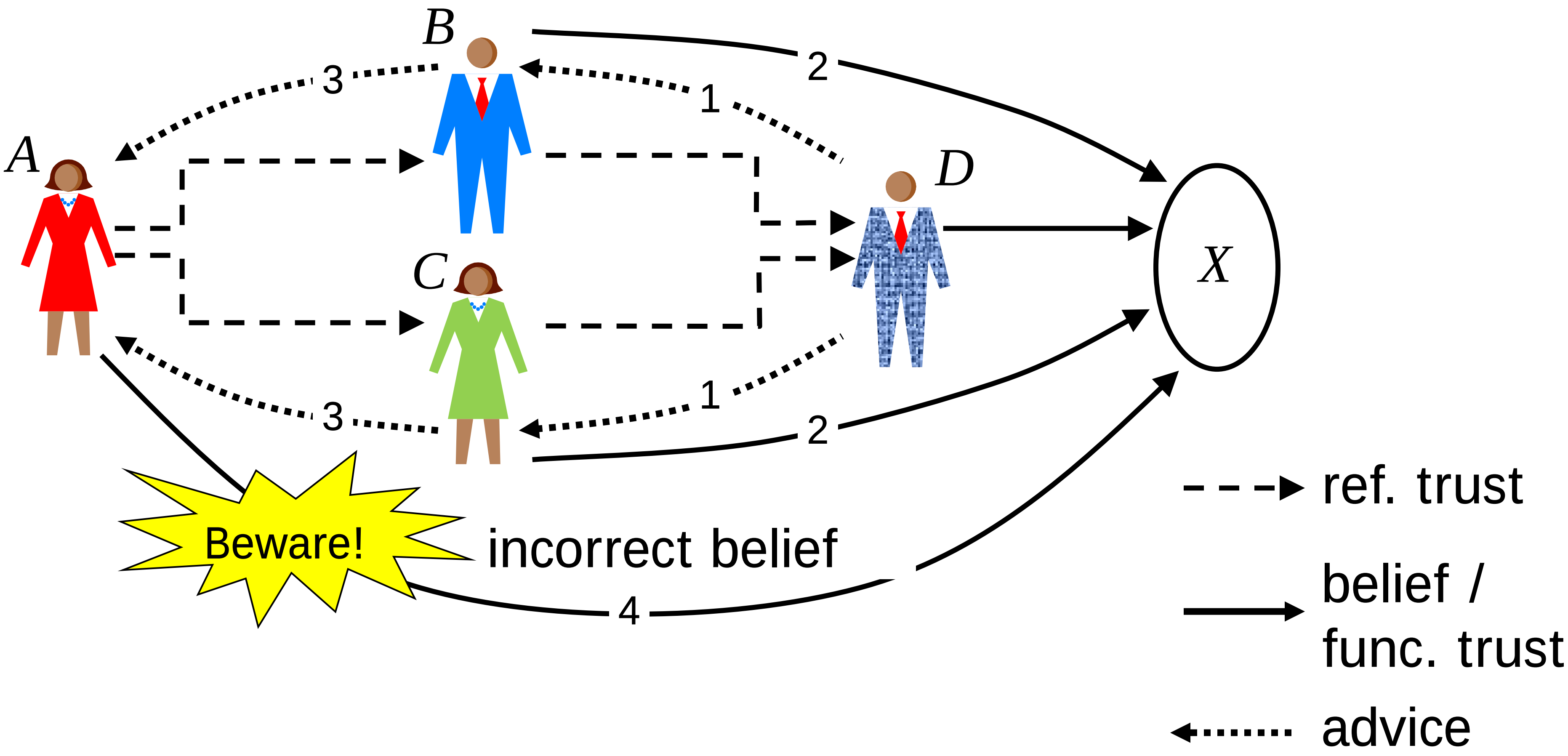
Discount and Fuse: Dilution and Confidence



Discounting dilutes trust confidence

Fusion strengthens trust confidence

Incorrect trust / belief derivation

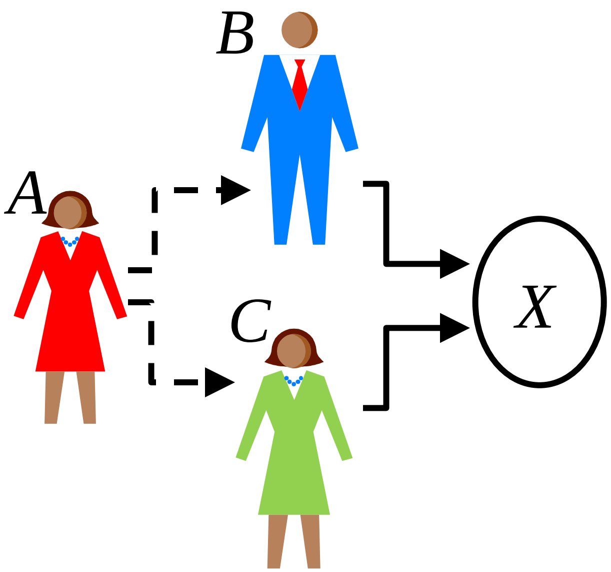


Perceived: $([A, B] : [B, X]) \lozenge ([A, C] : [C, X])$

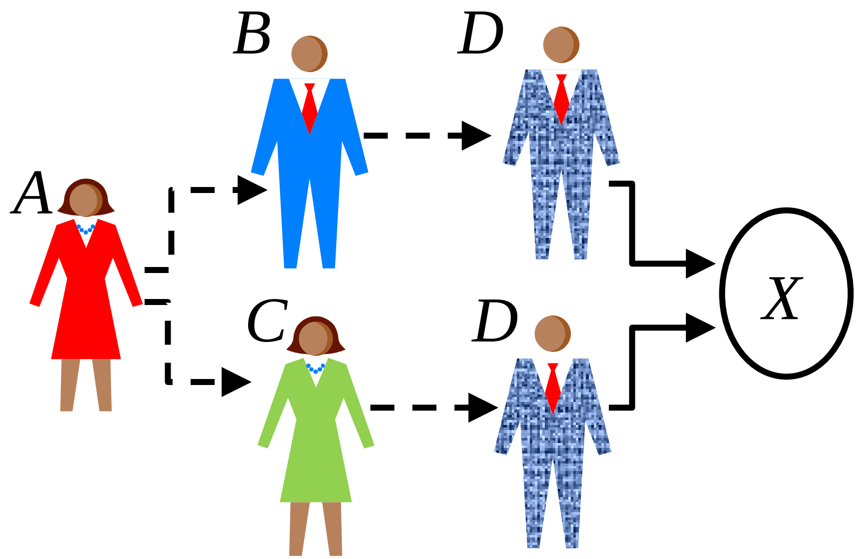
Hidden: $([A, B] : [B, D] : [D, X]) \lozenge ([A, C] : [C, D] : [D, X])$

Hidden and perceived topologies

Perceived topology:



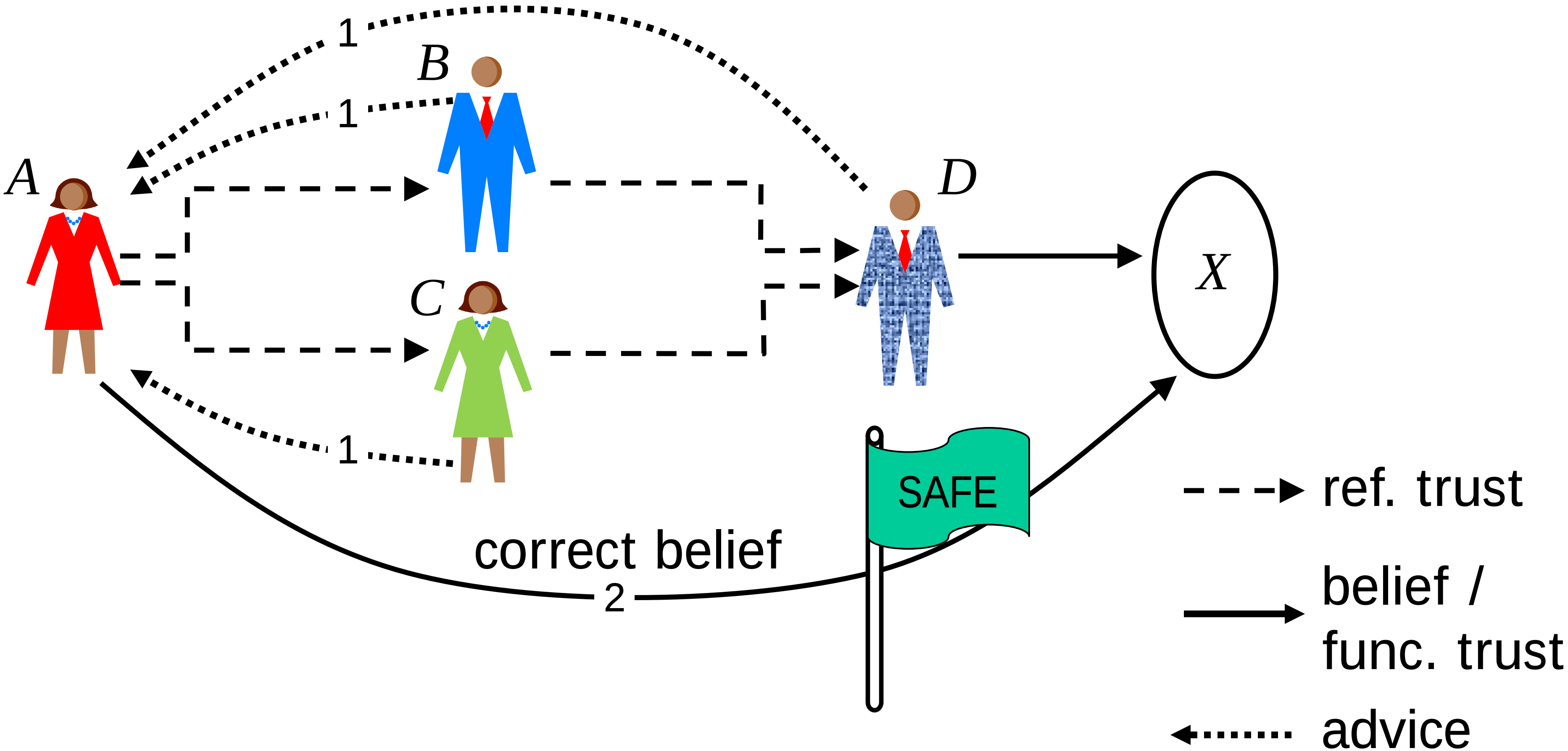
Hidden topology:



$$
\begin{aligned}
&([A, B] : [B, X]) \;\Diamond\; ([A, C] : [C, X]) \\
&\neq\; ([A, B] : [B, D] : \mathbf{[D, X]}) \;\Diamond\; ([A, C] : [C, D] : \mathbf{[D, X]})
\end{aligned}
$$

(D, E) is taken into account twice

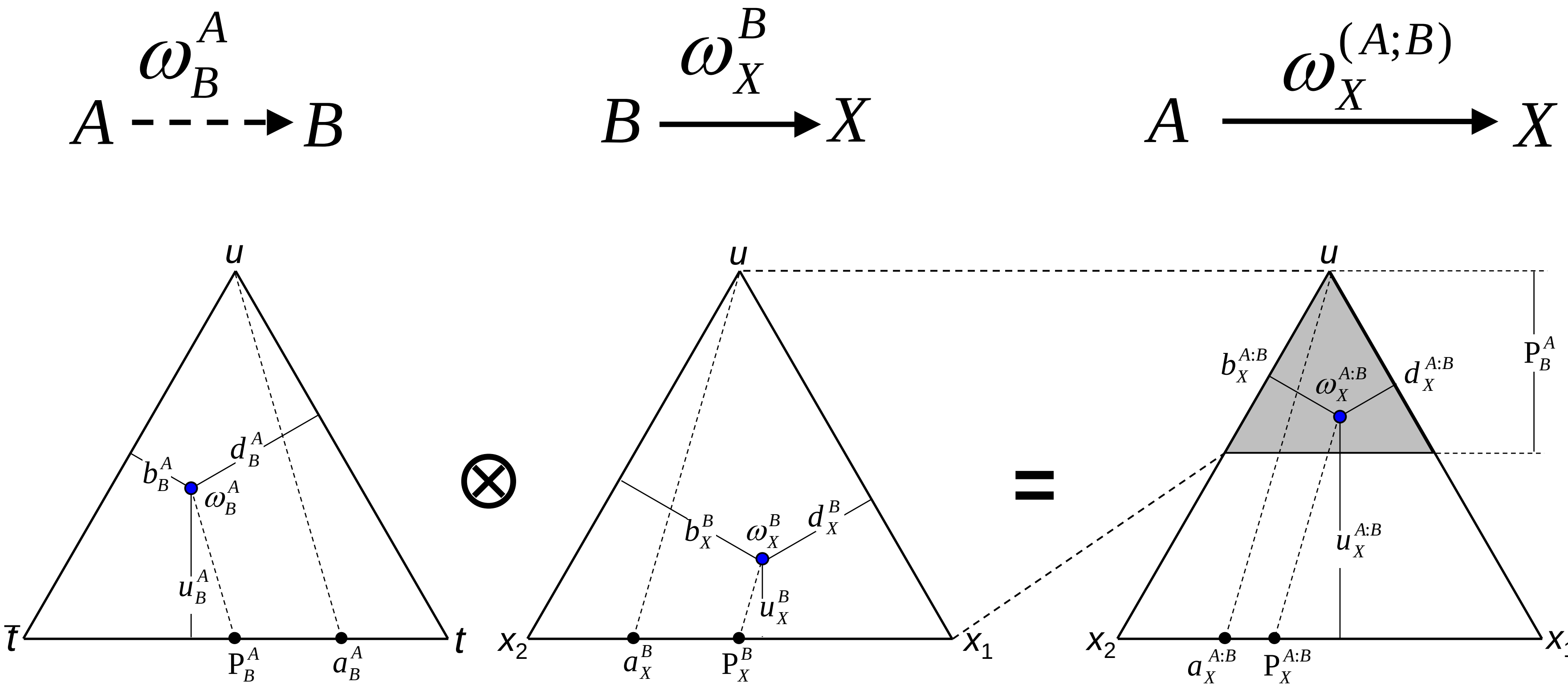
Correct trust / belief derivation



Perceived and_real
topologies are equal:

$$(([A;B]:[B;D]) \diamond ([A;C]:[C;D])):[D,X]$$

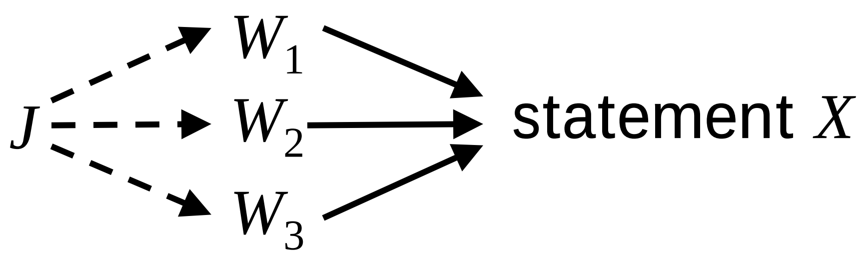
Computing discounted trust



A’s trust in *B* | *B*’s opinion about *X* | *A*’s derived opinion about *X*

Example: Weighing testimonies

- Computing beliefs about statements in court.
- J is the judge.
- W_1, W_2, W_3 are witnesses providing testimonies.



$$\omega_X^{(J;W_1)\Diamond(J;W_2)\Diamond(J;W_3)}$$

Computational trust with subjective logic

Trust Inference Demo - Microsoft Internet Explorer

File Edit View Favorites Tools Help

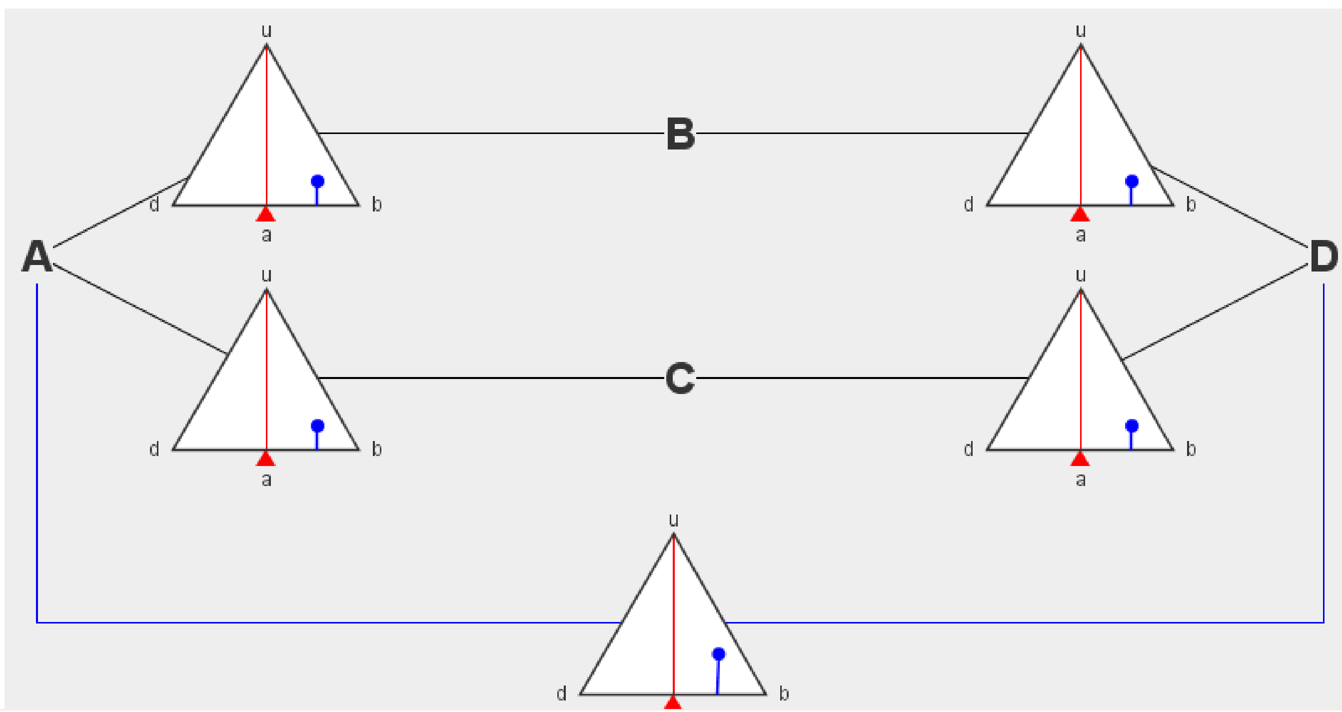
Address http://security.dstc.edu.au/spectrum/trustengine/demo2.html Go Links

Simple Trust Network Demo

Four entities, labelled A, B, C and D have opinios about each other represented as points in triangles. Entity A is trying to form an opinion about D, and receives opinions from B and C as to the trustworthiness of D. Furthermore, A has his own opinions about the trustworthiness of B and C.

SPECTRUM

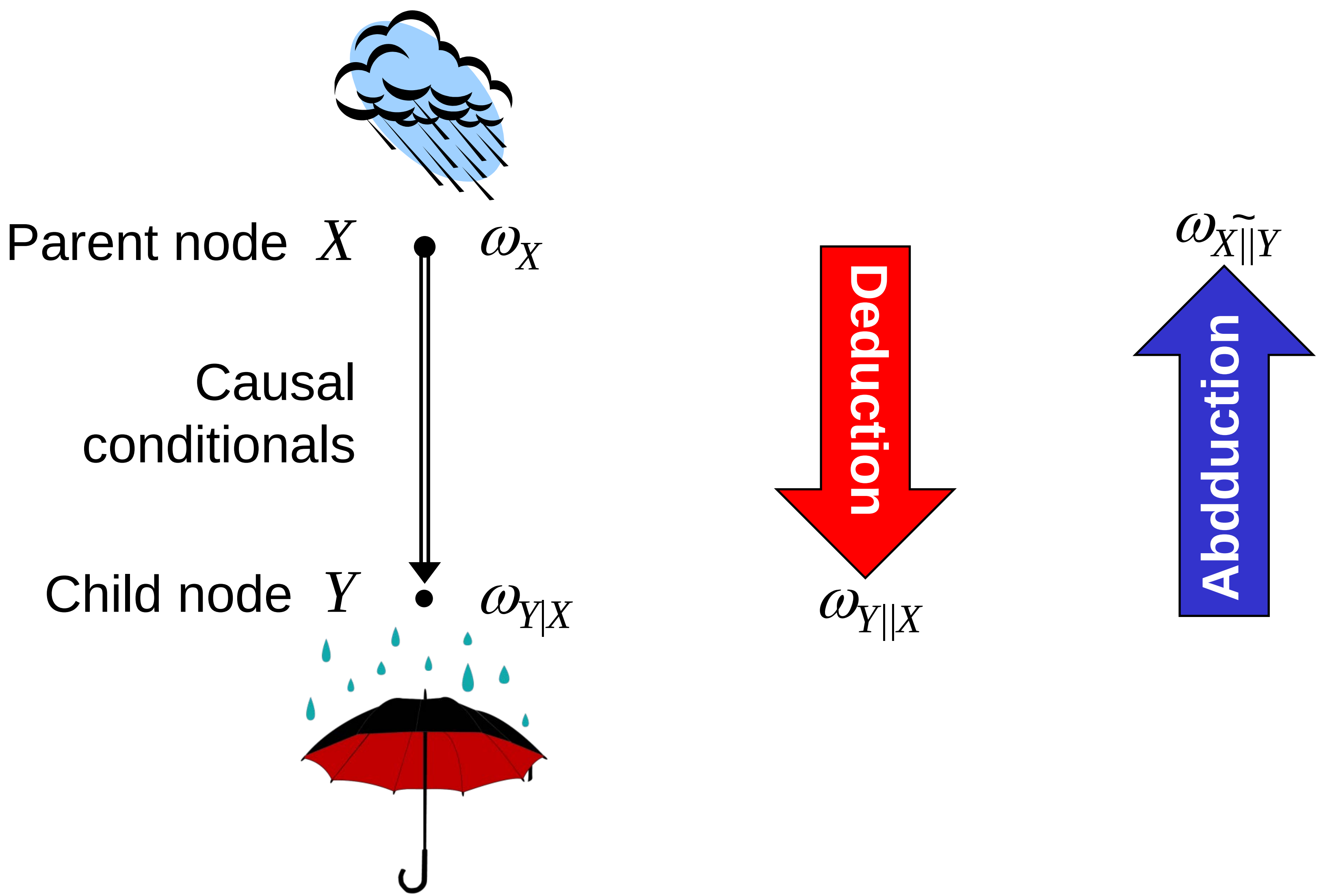
Left-click and drag opinion points to set opinion values. Entity A combines these opinions using the Subjective Logic Operators to derive his own opinion about D, as shown by the bottom opinion triangle. In detail, entity A *discounts* B's opinion about D by his opinion about B, and does similarly for C. Finally, he combines the two discounted opinions using the *consensus* operator in order to determine his opinion about D. Right-click on the opinion triangles to see the exact values of each opinion. Opinion values can also be visualised using three-coloured rectangles.



http://folk.uio.no/josang/sl/

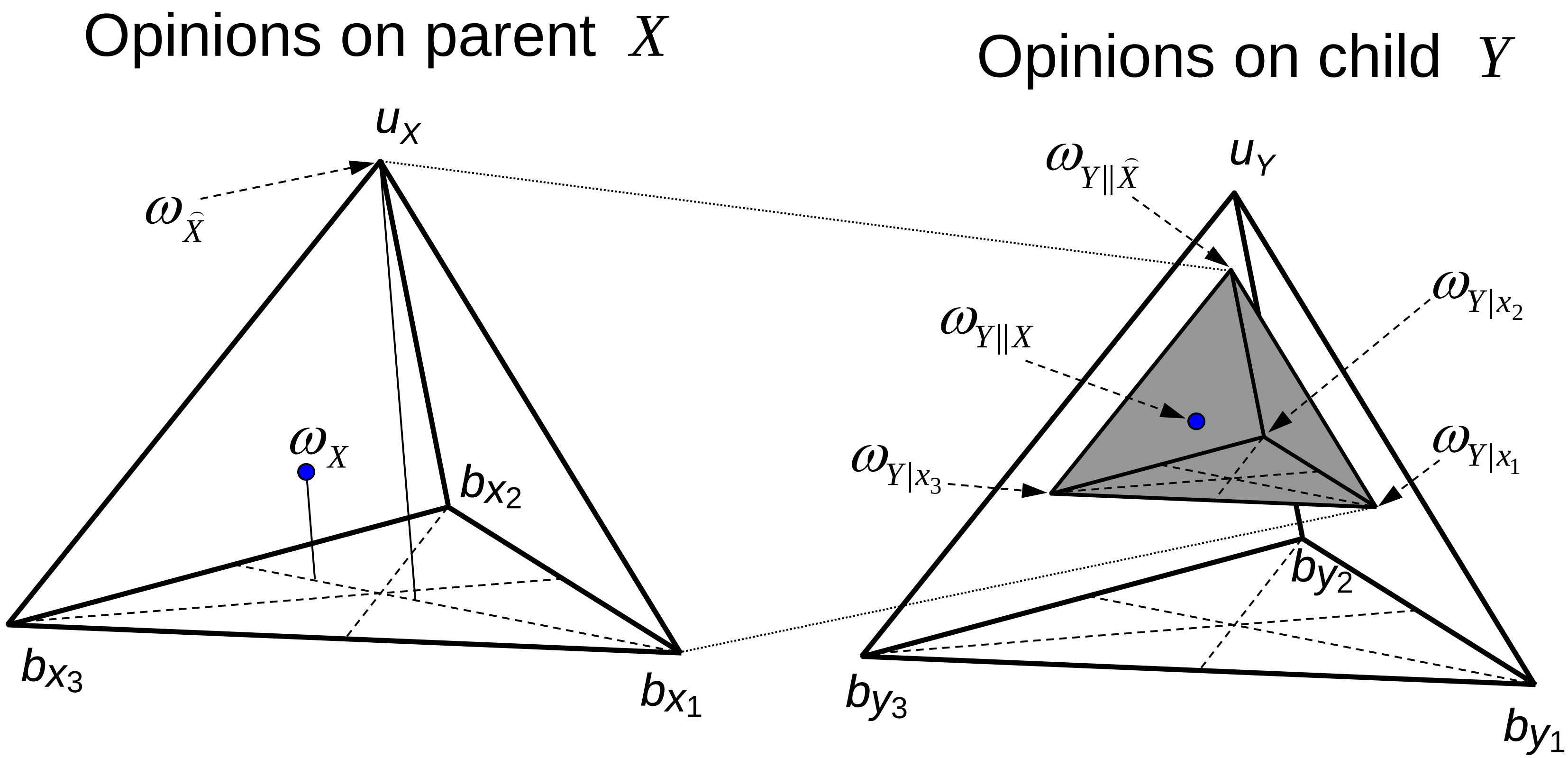
Bayesian Reasoning

Deduction and Abduction

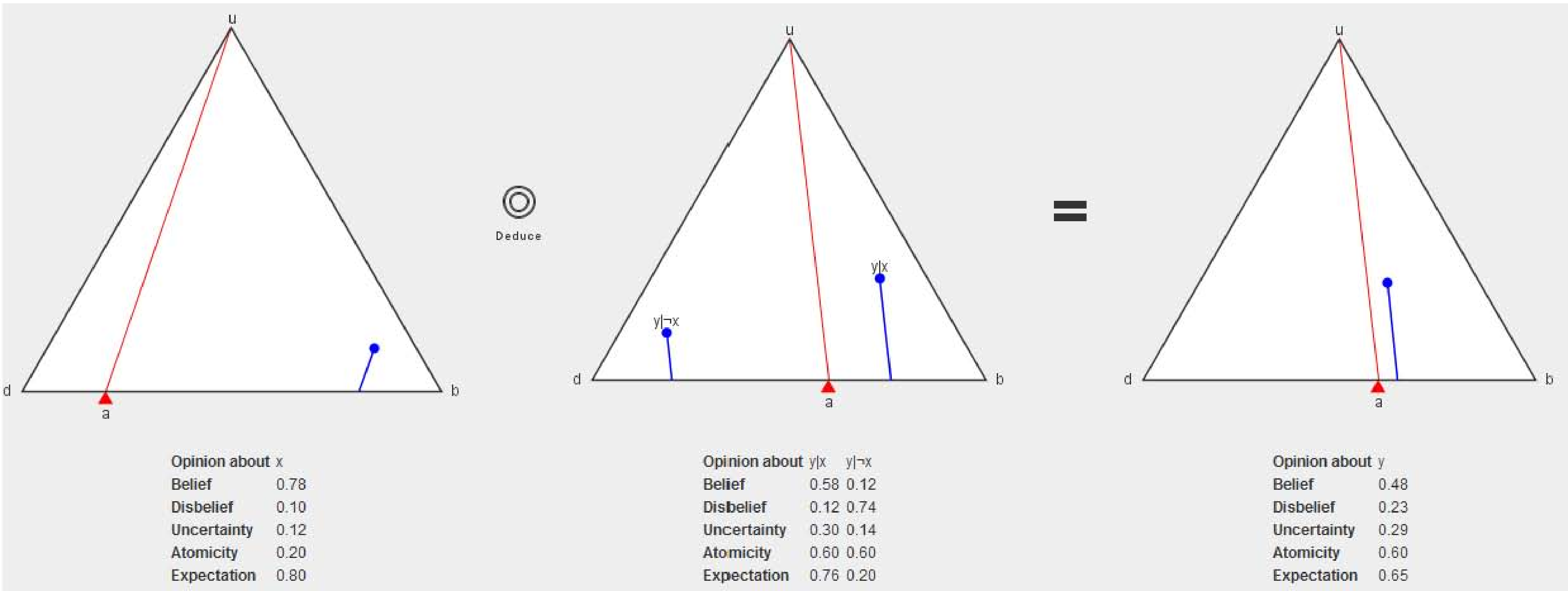


Deduction visualisation

- Evidence pyramid is mapped inside hypothesis pyramid as a function of the conditionals.
- Conclusion opinion is linearly mapped



Deduction – online operator demo



http://folk.uio.no/josang/sl/

Bayes' Theorem

- Traditional statement of Bayes' theorem:

$$p(x \mid y) = \frac{p(x)\,p(y|x)}{p(y)}$$

- Bayes' theorem with base rates:

$$p(x \mid y) = \frac{a(x)\,p(y|x)}{a(y)}$$

- Marginal base rates:

$$a(y) = a(x)\,p(y \mid x) + a(\overline{x})\,p(y \mid \overline{x})$$

- Bayes' theorem with marginal base rates

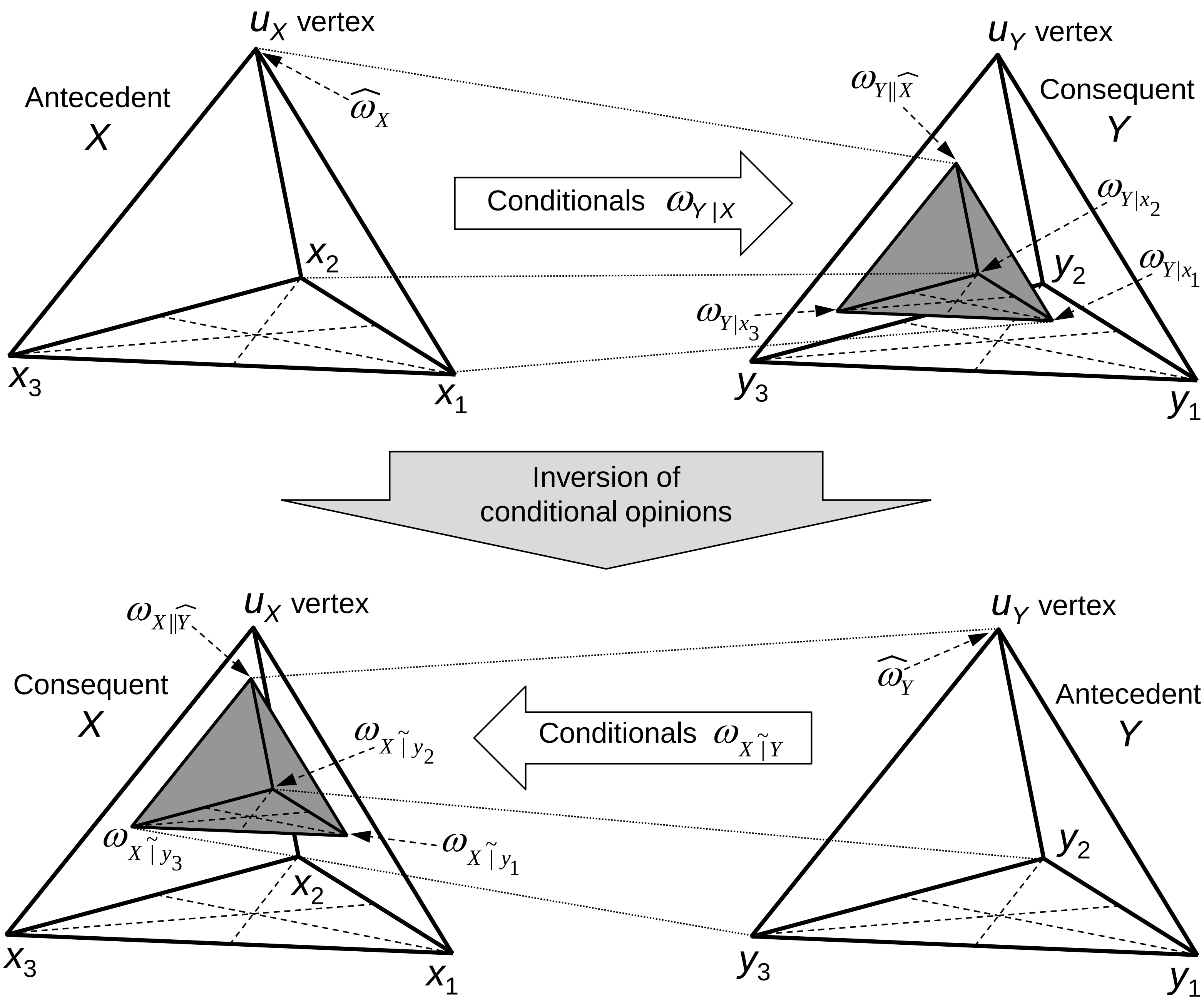
$$\begin{cases} p(x \mid y) = \dfrac{a(x)\,p(y|x)}{a(x)\,p(y|x)+a(\overline{x})\,p(y|\overline{x})} \\[2ex] p(x \mid \overline{y}) = \dfrac{a(x)\,p(\overline{y}|x)}{a(x)\,p(\overline{y}|x)+a(\overline{x})\,p(\overline{y}|\overline{x})} \end{cases}$$

The Subjective Bayes' Theorem

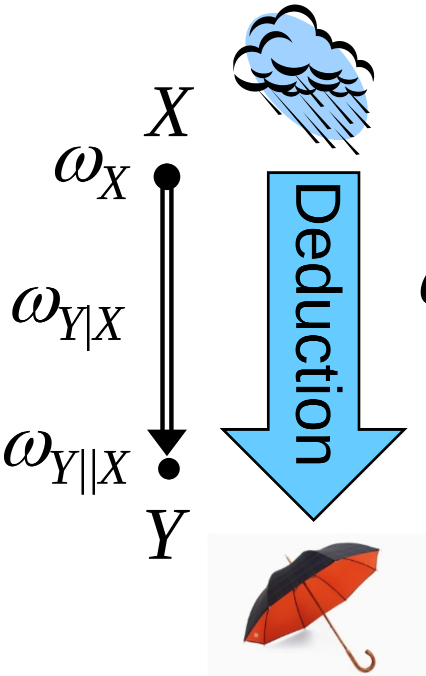
Binomial: $$\begin{array}{c} X \\ \Uparrow \\ Y \end{array} \quad (\omega_{x \tilde{|} y},\ \omega_{x \tilde{|} \overline{y}}) = \tilde{\phi}\,(\omega_{y|x},\ \omega_{y|\overline{x}},\ a_x) \quad \begin{array}{c} X \\ \Downarrow \\ Y \end{array}$$

Multinomial: $$\begin{array}{c} X \\ \Uparrow \\ Y \end{array} \quad \omega_{X \tilde{|} Y} = \tilde{\phi}\,(\omega_{Y|X},\ a_x) \quad \begin{array}{c} X \\ \Downarrow \\ Y \end{array}$$

Inversion Visualisation (Subjective Bayes' theorem)

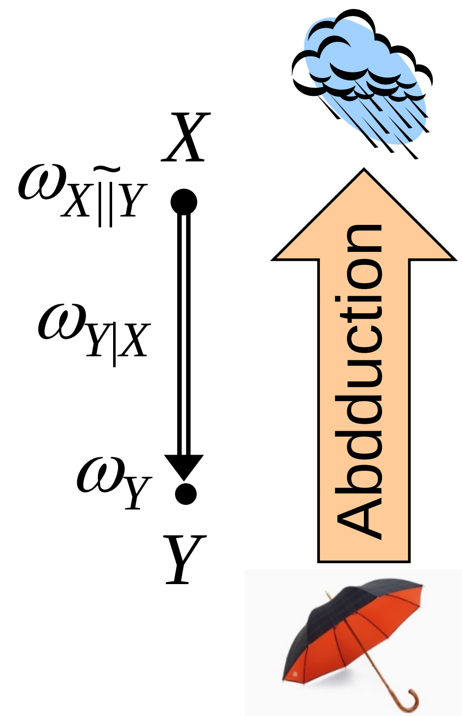


Deduction and abduction notation



$$\omega_{Y\|X} = \omega_X \circledcirc \omega_{Y|X}$$

$\omega_{Y|X}$



$$\begin{aligned}\omega_{X\widetilde{\|}Y} &= \omega_Y \,\widetilde{\circledcirc}\, (\omega_{Y|X},\ a_X) \\ &= \omega_Y \circledcirc \widetilde{\phi}\,(\omega_{Y|X},\ a_X) \\ &= \omega_Y \circledcirc \omega_{X\widetilde{|}Y}\end{aligned}$$

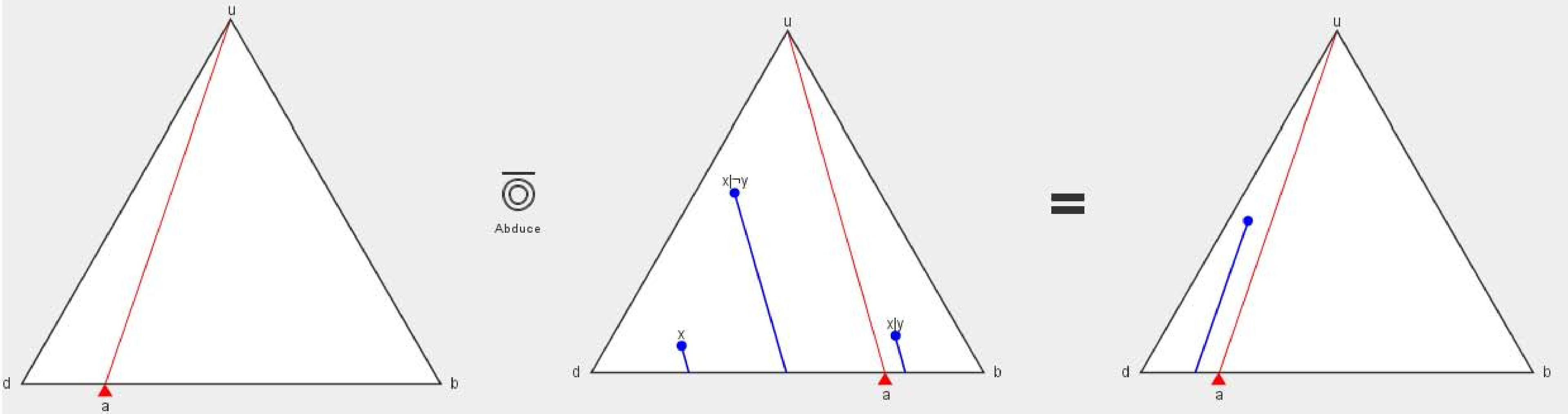
Example: Medical reasoning

- Medical test reliability determined by:
  - true positive rate $p(y \mid x)$ where x: infected
  - false positive rate $p(y \mid \bar{x})$ y: positive test
- Bayes' theorem: $p(x \mid y) = \dfrac{p(x)\,p(y \mid x)}{p(y)} = \dfrac{a(x)\,p(y \mid x)}{a(x)\,p(y \mid x) + a(\bar{x})\,p(y \mid \bar{x})}$
- Probabilistic model hides uncertainty
- Use subjective Bayes' theorem to determine $\omega_{(\text{infected})}$

$$\omega_{X \tilde{|} Y} = \tilde{\phi}\,(\omega_{Y|X},\ a_X)$$

- GP derives $\omega_{(\text{infected}\,\tilde{|}\,\text{positive})}$ and $\omega_{(\text{infected}\,\tilde{|}\,\text{negative})}$
- Finally compute diagnosis $\omega_{(\text{infected}\,\tilde{\|}\,\text{test result})}$
- Medical reasoning with SL reflects uncertainty

Abduction – Online operator demo



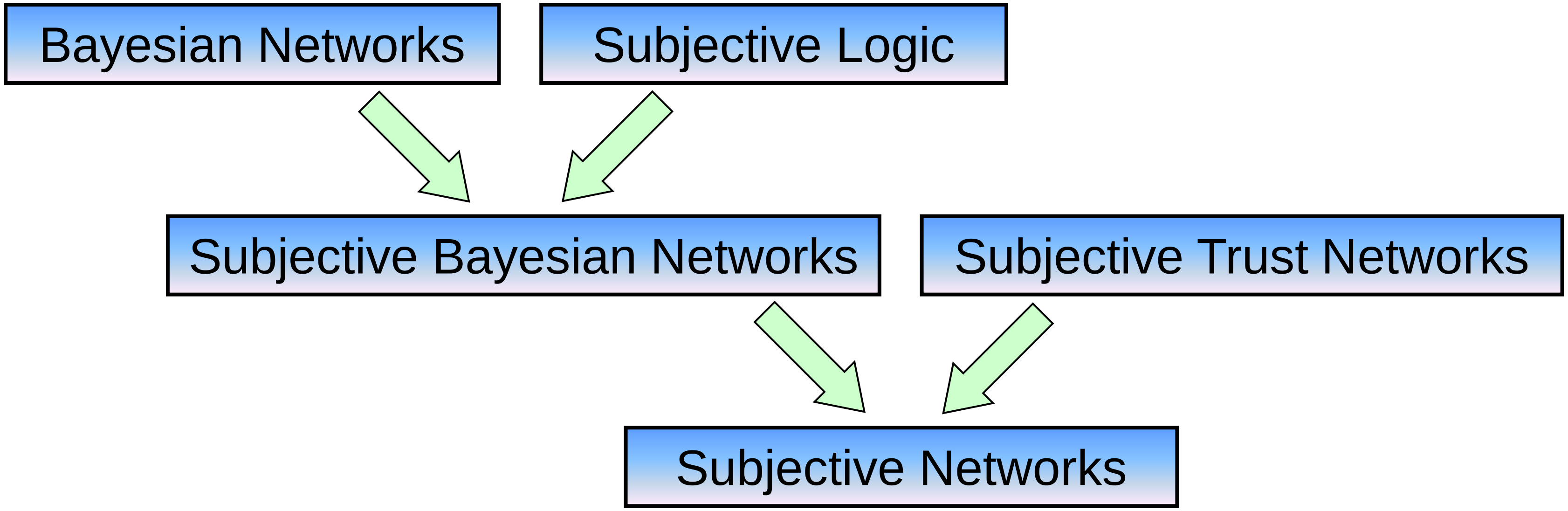
Opinion about	br(y)
Belief	0.00
Disbelief	0.00
Uncertainty	1.00
Atomicity	0.20
Expectation	0.20

Opinion about	x	x|y	x|¬y
Belief	0.19	0.72	0.10
Disbelief	0.73	0.17	0.37
Uncertainty	0.08	0.11	0.53
Atomicity	0.75	0.75	0.75
Expectation	0.25	0.80	0.50

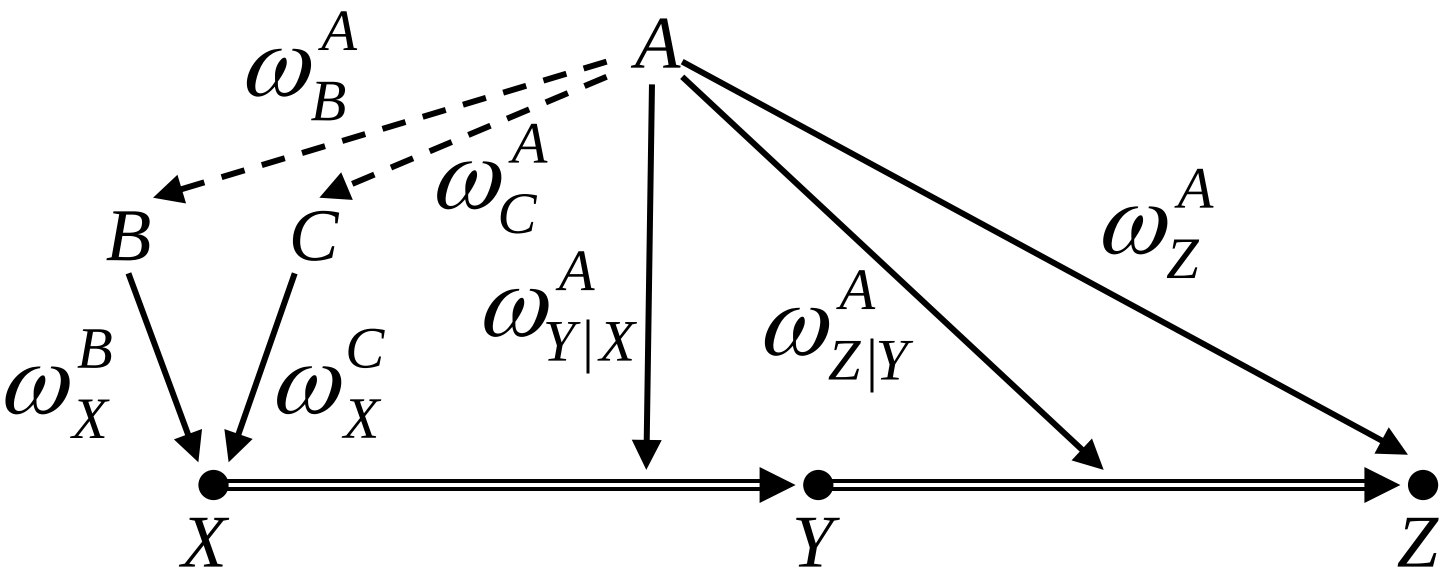
Opinion about	y
Belief	0.05
Disbelief	0.50
Uncertainty	0.45
Atomicity	0.20
Expectation	0.14

http://folk.uio.no/josang/sl/

The General Idea of Subjective Networks

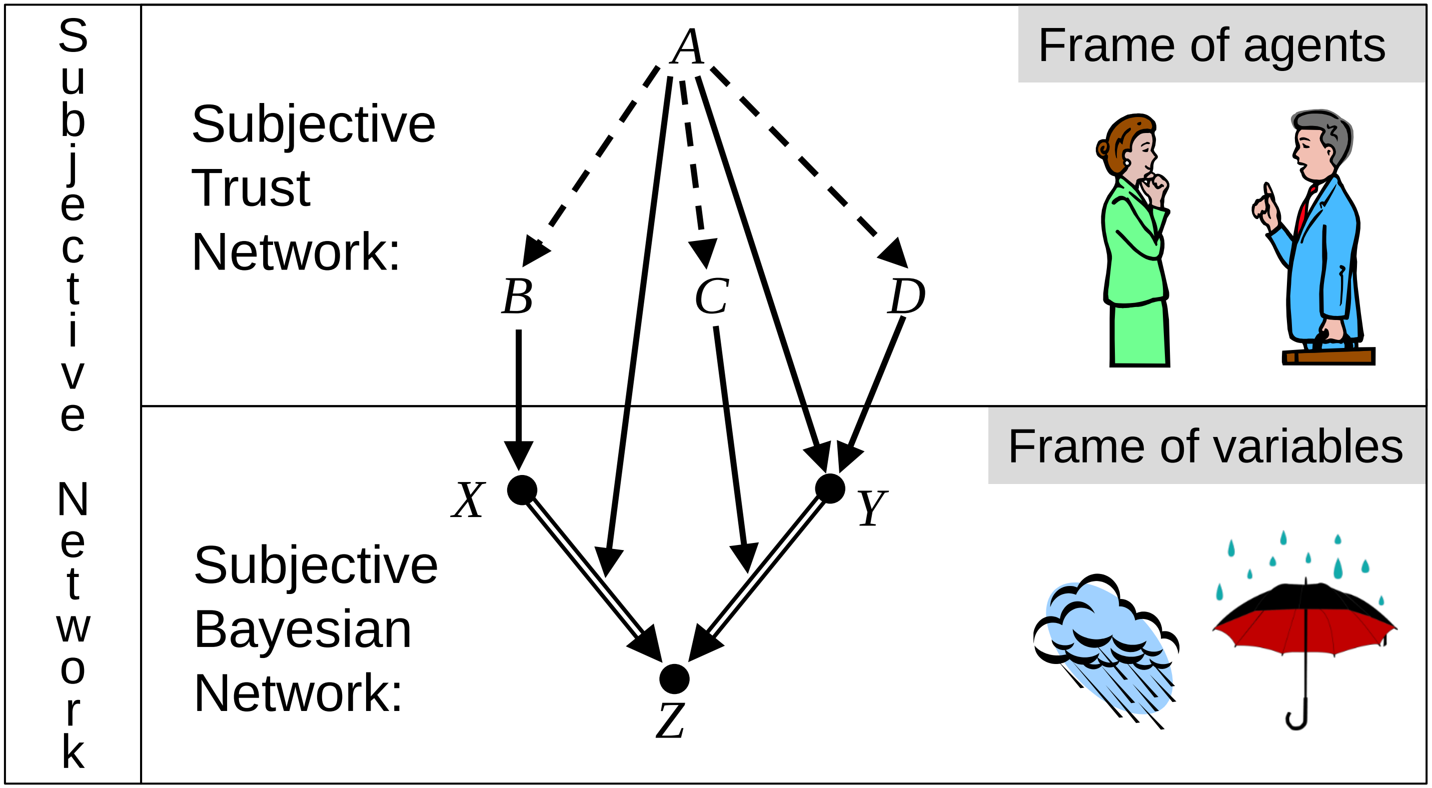


Example SN Model



$$\omega_Z^A = (((\omega_B^A \otimes \omega_X^B) \oplus (\omega_C^A \otimes \omega_X^C)) \circledcirc \omega_{Y|X}^A) \circledcirc \omega_{Z|Y}^A$$

Subjective Networks



Legend:

Trust relationship

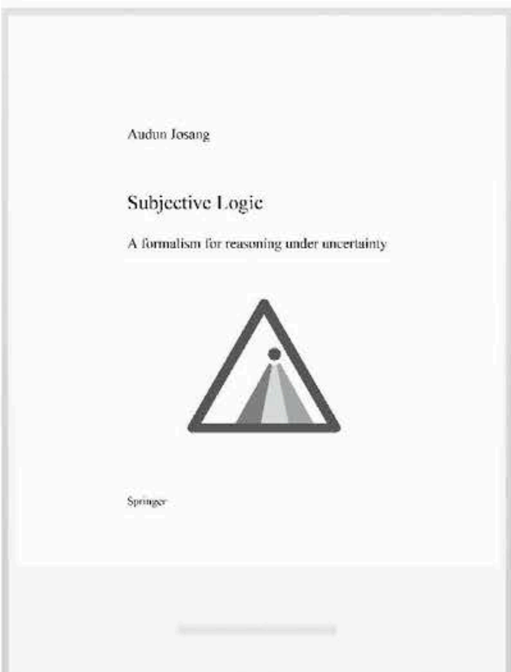
Belief relationship

Conditional relationship

New Book on Subjective Logic

Springer

springer.com



A. Jøsang

Subjective Logic

A Formalism for Reasoning Under Uncertainty

Series: Artificial Intelligence: Foundations, Theory, and Algorithms

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- **Helpful for researchers and practitioners who want to build artificial reasoning models and tools for solving real-world problems**

This is the first comprehensive treatment of subjective logic and all its operations. The author developed the approach, and in this book he first explains subjective opinions,