APPENDIX: SUPPLEMENTARY MATERIAL
Characterizing Tightness of LP Relaxations by Forbidding Signed Minors

Here we expand on the observation in §6 of the main paper that LP+TRI can sometimes perform worse as singleton potential strengths increase. We also observe the power of Theorem 14 beyond Theorems 10 or 13.

It is easily seen that if all singleton potentials are sufficiently strong relative to edge potentials, then for each variable, local considerations will be sufficient to determine its MAP configuration, and hence any LP+$\mathcal{L}_r$ relaxation (even on LOC) will be tight. For LP+LOC: if there is a frustrated cycle then stronger edge potentials typically lead to a worse approximation; stronger singleton potentials typically lead to a better approximation.

By examining the semimetric polytope MET, which is equivalent to TRI, we noted just before §5.2 that TRI may be considered *universally rooted*, and hence, in contrast to LOC, TRI treats singleton and edge potentials in an elegantly symmetric way.

In Figure 5, we provide an illustration of this effect. We consider the fully connected model on 4 variables with $W_{ij} = -1 \ \forall (i,j) \in E$ as all θ_i potentials are varied together. The signed graph G is an odd-K_4. From Theorem 14, we know that LP+TRI will be tight if there is no odd-K_5 minor in ∇G. In particular, it must be tight if all singleton potentials are 0 (it must also be tight if any of the singleton potentials were to take opposite signs since this avoids an odd-K_5; this demonstrates the power of Theorem 14 beyond Theorem 10 or Theorem 13).

As θ_i moves away from 0, we obtain an odd-K_5 and the error of LP+TRI increases. Note that the error is symmetric for θ_i on either side of 0, which may be understood by seeing that in the signed suspension graph ∇G, we may resign by flipping the 0 node, which exactly flips the signs of all θ_i potentials; see 3.1.

Developing a better understanding of how approximation error varies with potential strengths is a promising area for future research.

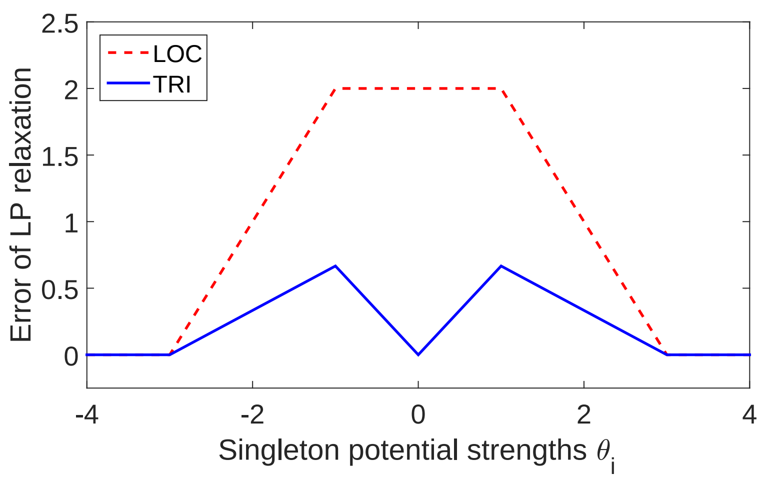

Figure 5: Error of the LP+LOC and LP+TRI relaxations (optimum score minus the optimum integral score) for a fully connected model on 4 variables with $W_{ij} = -1 \ \forall (i,j) \in E$. All θ_i singleton potentials take the same value, which is varied as shown. For sufficiently strong singleton potentials, both LP+LOC and LP+TRI are tight. Starting from 0 singleton potentials, LP+LOC performs monotonically better as singleton potentials strengthen. LP+TRI, in contrast, is tight for 0 singleton potentials, then gets worse and then better again as singleton potentials strengthen.