A Appendix

A.1 Proof of Lemma 1

Proof. Fix a target n, for any resource k, we have:

$$\begin{aligned}\bar{\mathbf{c}}_n(\mathcal{D}_\theta) &= \mathbb{E}_{s\sim\mathcal{D}_\theta}[\mathbf{c}_n(\mathbf{s})] = \mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(\mathbf{c}_n(\mathbf{s}) = 1) \\ &= \mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(\mathbf{c}_n(\mathbf{s}) = 1 \mid c_{-k,n}(\mathbf{s}) = 0)\mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 0) \\ &+ \mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(\mathbf{c}_n(\mathbf{s}) = 1 \mid c_{-k,n}(\mathbf{s}) = 1)\mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 1) \\ &= \mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(\mathbf{c}_n(\mathbf{s}) = 1 \mid c_{-k,n}(\mathbf{s}) = 0)\mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 0) \\ &+ \mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 1)\end{aligned}$$

For the same resource k and any schedule l, observe that $\frac{\partial}{\partial\theta_{k,l}}\bar{\mathbf{c}}_n(\mathcal{D}_\theta)$ is therefore equal to:

$$\begin{aligned}&\frac{\partial}{\partial\theta_{k,l}}\mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(\mathbf{c}_n(\mathbf{s}) = 1 \mid c_{-k,n}(\mathbf{s}) = 0)\mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 0) \\ &\quad= \mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 0)\frac{\partial}{\partial\theta_{k,l}}\sum_{l'=0}^{d_k} c_{k,l',n}\theta_{k,l'} \\ &\quad= (c_{k,l,n} - c_{k,0,n})\,\mathbb{P}_{\mathbf{s}\sim\mathcal{D}_\theta}(c_{-k,n}(\mathbf{s}) = 0).\end{aligned}$$

□

A.2 Proof of Theorem 2

Proof. Given a parameterization θ, denote the probability that agent r_k is assigned to some schedule $\mathbf{s}_k = [t_{n_{k,1}}, \ldots, t_{n_{k,L}}]$ by $p(\mathbf{s}_k \mid \theta)$. Fixing a parameter $\mathbf{w}_{k,n}$, and subsequence of length $l-1$, we can apply equation 6, to conclude that:

$$\begin{aligned}&\frac{\partial}{\partial\mathbf{w}_{k,n}}\mathbb{P}[\mathbf{s}_k(l) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta] \\ &\quad= \frac{\partial}{\partial\mathbf{w}_{k,n}}\frac{\exp(\mathbf{w}_{k,n_{k,l}})}{\sum_{t_{n'}\in F(\mathbf{s}_{k,1:(l-1)})}\exp(\mathbf{w}_{k,n'})} \\ &\quad= \mathbf{1}[n = n_{k,l}]\frac{\exp(\mathbf{w}_{k,n_{k,l}})}{\sum_{t_{n'}\in F(\mathbf{s}_{k,1:(l-1)})}\exp(\mathbf{w}_{k,n'})} \\ &\quad- \frac{\exp(\mathbf{w}_{k,n_{k,l}})\frac{\partial}{\partial\mathbf{w}_{k,n}}\sum_{t_{n'}\in F(\mathbf{s}_{k,1:(l-1)})}\exp(\mathbf{w}_{k,n'})}{\left(\sum_{t_{n'}\in F(\mathbf{s}_{k,1:(l-1)})}\exp(\mathbf{w}_{k,n'})\right)^2} \\ &\quad= \mathbf{1}[n = n_{k,l}]\mathbb{P}[\mathbf{s}_k(i) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta] \\ &\quad- \frac{\exp(\mathbf{w}_{k,n_{k,l}})\mathbf{1}[t_n \in F(\mathbf{s}_{k,1:(l-1)})]\exp(\mathbf{w}_{k,n})}{\left(\sum_{t_{n'}\in F(\mathbf{s}_{k,1:(l-1)})}\exp(\mathbf{w}_{k,n'})\right)^2} \\ &\quad= \mathbb{P}[\mathbf{s}_k(l) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta]\mathbf{1}[n = n_{k,l}] \\ &\quad- \mathbb{P}[\mathbf{s}_k(l) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta]\mathbb{P}[\mathbf{s}_k(l) = t_n \mid \mathbf{s}_{k,1:(l-1)}, \theta]\end{aligned}$$

Equation 6 also tells us that:

$$p(\mathbf{s}_k \mid \theta) = \prod_{l=1}^{L}\mathbb{P}[\mathbf{s}_k(l) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta]$$

Applying the chain rule gives us:

$$\begin{aligned}\frac{\frac{\partial}{\partial\mathbf{w}_{k,n}}p(\mathbf{s}_k \mid \theta)}{p(\mathbf{s}_k \mid \theta)} &= \sum_{l=1}^{L}\frac{\frac{\partial}{\partial\mathbf{w}_{k,n}}\mathbb{P}[\mathbf{s}_k(l) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta]}{\mathbb{P}[\mathbf{s}_k(l) = t_{n_{k,l}} \mid \mathbf{s}_{k,1:(l-1)}, \theta]} \\ &= \sum_{l=1}^{L}\mathbf{1}[n = n_{k,l}] - \mathbb{P}[\mathbf{s}_k(l) = t_n \mid \mathbf{s}_{k,1:(l-1)}, \theta]\end{aligned}$$

Borrowing equation 5 from the proof of Theorem 1, concludes the proof. □