REASONING UNDER UNCERTAINTY WITH SUBJECTIVE LOGIC

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About me

- Prof. Audun Jøsang, University of Oslo, 2008
- Research interests
  - Information Security
  - Bayesian Reasoning
- Bio
  - MSc Telecom, NTH, 1988
  - Engineer, Alcatel Telecom
  - MSc Security, London 1993
  - PhD Security, NTNU 1998
  - A.Prof. QUT, Australia, 2000
Tutorial overview

1. Representations of subjective opinions

2. Operators of subjective logic

3. Applications of subjective logic:
   - Trust fusion and transitivity
   - Trust networks
   - Bayesian reasoning
   - Subjective networks
The General Idea of Subjective Logic

- Logic
- Probability

Probabilistic Logic

Uncertainty & Subjectivity

Subjective Logic
# Probabilistic Logic Examples

<table>
<thead>
<tr>
<th>Binary Logic</th>
<th>Probabilistic logic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AND:</strong> $x \land y$</td>
<td>$p(x \land y) = p(x)p(y)$</td>
</tr>
<tr>
<td><strong>OR:</strong> $x \lor y$</td>
<td>$p(x \lor y) = p(x) + p(y) - p(x)p(y)$</td>
</tr>
<tr>
<td><strong>MP:</strong> ${ x \rightarrow y, x }$ $\Rightarrow y$</td>
<td>$p(y) = p(x)p(y</td>
</tr>
<tr>
<td><strong>MT:</strong> ${ x \rightarrow y, \overline{y} }$ $\Rightarrow \overline{x}$</td>
<td></td>
</tr>
</tbody>
</table>

$$p(x | y) = \frac{a(x)p(y | x)}{a(x)p(y | x) + a(\overline{x})p(y | \overline{x})}$$

$$p(x | \overline{y}) = \frac{a(x)p(\overline{y} | x)}{a(x)p(\overline{y} | x) + a(\overline{x})p(\overline{y} | \overline{x})}$$

$$p(x) = p(y)p(x | y) + p(\overline{y})p(x | \overline{y})$$
Probability and Uncertainty

Frequentist (aleatory):
• Confident when based on much observation evidence
• Unconfident when based on little observation evidence
• E.g.: Probability of heads when flipping coin is $\frac{1}{2}$ and confident

Subjective (epistemic):
• Confident when dynamics of situation are known
• Unconfident when dynamics of situation are unknown
• E.g. Probability of Oswald killed Kennedy is $\frac{1}{2}$ but unconfident
Domains, variables and opinions

Binary domain $X = \{x, \bar{x}\}$

Binary variable $X = x$

Binomial opinion

3-ary domain $X$

Random variable $X \in X$

Multinomial opinion

Hyperdomain $\mathcal{R}(X)$

Hypervariable $X \in \mathcal{R}(X)$

Hypernomial opinion
Domains and Hyperdomains

- A domain $X$ is a state space of distinct possibilities
- Powerset $\mathcal{P}(X) = 2^X$, set of subsets, including $\{X, \emptyset\}$
- Reduced powerset $\mathcal{R}(X) = \mathcal{P}(X) \setminus \{X, \emptyset\}$
- $\mathcal{R}(X) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$
- $\mathcal{R}(X)$ called Hyperdomain
- Cardinalities
  - $|X| = 3$ in this example
  - $|\mathcal{P}(X)| = 2^{|X|}$
    - $= 8$ in this example
  - $|\mathcal{R}(X)| = 2^{|X|} - 2$
    - $= 6$ in this example
<table>
<thead>
<tr>
<th>Domain</th>
<th>Binomial Opinion</th>
<th>Multinomial Opinion</th>
<th>Hypernomial Opinion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary domain</td>
<td>$X$</td>
<td>$X$</td>
<td>$\mathcal{R}(X)$</td>
</tr>
<tr>
<td>Binary variable</td>
<td>$X = x$</td>
<td>$X \in X$</td>
<td>$X \in \mathcal{R}(X)$</td>
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<table>
<thead>
<tr>
<th>Opinion representation</th>
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<tbody>
<tr>
<td><img src="image1.png" alt="Binomial Opinion" /></td>
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<tr>
<td><img src="image2.png" alt="Multinomial Opinion" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Hypernomial Opinion" /></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>PDF representation</th>
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</thead>
<tbody>
<tr>
<td>Beta PDF over $x$</td>
</tr>
<tr>
<td>Dirichlet PDF over $X$</td>
</tr>
<tr>
<td>Hyper Dirichlet over $X$</td>
</tr>
</tbody>
</table>

![Image](image.png)

*FIG 1: Beta function after 7 positive and 1 negative results*
Binomial subjective opinions

- Belief mass and base rate on binary domains
  - \( b^A_x = b(x) \) is observer \( A \)'s belief in \( x \)
  - \( d^A_x = b(\bar{x}) \) is observer \( A \)'s disbelief in \( x \)
  - \( u^A_x = b(X) \) is observer \( A \)'s uncertainty about \( x \)
  - \( a^A_x \) is the base rate of \( x \)

Binomial opinion

\[
\omega^A_x = (b^A_x, d^A_x, u^A_x, a^A_x)
\]

Base rate of \( x \)

Binary domain

\[
b^A_x + d^A_x + u^A_x = 1
\]
Binomial opinions

• Ordered quadruple:
  \( \omega_x = (b_x, d_x, u_x, a_x) \)
  – \( b_x \): belief
  – \( d_x \): disbelief
  – \( u_x \): uncertainty (vacuity of evidence)
  – \( a_x \): base rate

• \( b_x + d_x + u_x = 1 \)

• Projected probability: \( P(x) = b_x + a_x \cdot u_x \)

Example \( \omega_x = (0.4, 0.2, 0.4, 0.9) \), \( P(x) = 0.76 \)
Opinion types

Absolute opinion: $b_x = 1$. Equivalent to TRUE.

Dogmatic opinion: $u_x = 0$. Equivalent to probabilities.

Vacuous opinion: $u_x = 1$. Equivalent to UNDEFINED.

General uncertain opinion: $u_x \neq 0$. 
Beta PDF representation

\[ \text{Beta}(p(x), \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} p(x)^{\alpha-1}(1 - p(x))^{\beta-1} \]

\[ \alpha = r + Wa \]
\[ \beta = s + W(1-a) \]

\( r \): # observations of \( x \)

\( s \): # observations of \( \bar{x} \)

\( a \): base rate of \( x \)

\( W = 2 \): non-informative prior weight

\( E(x) \): Expected probability

\( E(x) = P(x) \)

Example: \( r = 2, \quad s = 1, \quad a = 0.9, \quad E(x) = 0.76 \)
Binomial Opinion ↔ Beta PDF

- \((r,s,a)\) represents Beta PDF evidence parameters.
- \((b,d,u,a)\) represents binomial opinion.
- \(P(x) = E(x)\)

\[
\begin{align*}
\begin{cases} 
  r &= \frac{Wb}{u} \\
  s &= \frac{Wd}{u} \\
  b + d + u &= 1
\end{cases}
\]

- \(W = 2\)

\[
\begin{align*}
  b &= \frac{r}{r+s+W} \\
  d &= \frac{s}{r+s+W} \\
  u &= \frac{W}{r+s+W}
\end{align*}
\]
Online demo

http://folk.uio.no/josang/sl/
### Likelihood and Confidence

#### Likelihood categories:
- Absolutely not
- Very unlikely
- Unlikely
- Somewhat unlikely
- Chances about even
- Somewhat likely
- Likely
- Very likely
- Absolutely

#### Confidence categories:
- No confidence
- Low confidence
- Some confidence
- High confidence
- Total confidence

<table>
<thead>
<tr>
<th>Confidence categories:</th>
<th>Absolutely not</th>
<th>Very unlikely</th>
<th>Unlikely</th>
<th>Somewhat unlikely</th>
<th>Chances about even</th>
<th>Somewhat likely</th>
<th>Likely</th>
<th>Very likely</th>
<th>Absolutely</th>
</tr>
</thead>
<tbody>
<tr>
<td>No confidence</td>
<td>E</td>
<td>9E</td>
<td>8E</td>
<td>7E</td>
<td>6E</td>
<td>5E</td>
<td>4E</td>
<td>3E</td>
<td>2E</td>
</tr>
<tr>
<td>Low confidence</td>
<td>D</td>
<td>9D</td>
<td>8D</td>
<td>7D</td>
<td>6D</td>
<td>5D</td>
<td>4D</td>
<td>3D</td>
<td>2D</td>
</tr>
<tr>
<td>Some confidence</td>
<td>C</td>
<td>9C</td>
<td>8C</td>
<td>7C</td>
<td>6C</td>
<td>5C</td>
<td>4C</td>
<td>3C</td>
<td>2C</td>
</tr>
<tr>
<td>High confidence</td>
<td>B</td>
<td>9B</td>
<td>8B</td>
<td>7B</td>
<td>6B</td>
<td>5B</td>
<td>4B</td>
<td>3B</td>
<td>2B</td>
</tr>
<tr>
<td>Total confidence</td>
<td>A</td>
<td>9A</td>
<td>8A</td>
<td>7A</td>
<td>6A</td>
<td>5A</td>
<td>4A</td>
<td>3A</td>
<td>2A</td>
</tr>
</tbody>
</table>
Mapping qualitative to opinion

- Category mapped to corresponding field of triangle
- Mapping depends on base rate
- Non-existent categories depending on base-rates

\[ a = \frac{1}{3} \]

\[ a = \frac{2}{3} \]
Mapping categories to opinions

- Overlay category matrix with opinion triangle
- Matrix skewed as a function of base rate
- Not all categories map to opinions
  - For a low base rate, it is impossible to describe an event as highly likely and uncertain, but possible to describe it as highly unlikely and uncertain.
  - E.g. with regard to tuberculosis which has a low base rate, it would be wrong to say that a patient is likely to be infected, with high uncertainty. Similarly it would be possible to say that the patient is probably not infected, with high uncertainty
Multinominal domain

• Generalisation of binary domain

• Set of exclusive and exhaustive singletons.

• Example domain: $X=\{x_1, x_2, x_3, x_4\}$, $|X|=4$. 
Multinomial Opinions

- Domain: $X = \{x_1 \ldots x_k\}$
- Random variable $X \in X$
- Multinomial opinion: $\omega_X = (b_X, u_X, a_X)$

- Belief mass distribution $b_X$ where $u + \Sigma b_X(x) = 1$
  $b_X(x)$ is belief mass on $x \in X$
- Uncertainty mass: $u_X$ is a single value in range $[0,1]$
- Base rate distribution $a_X$ where $\Sigma a_X(x) = 1$
  $a_X(x)$ is base rate of $x \in X$
- Projected probability: $P_X(x) = b_X(x) + a_X(x) \cdot u_X$
Dirichlet PDF representation

$$\text{Dir}(p_X) = \frac{\Gamma \left( \sum_{i=1}^{k} \alpha_X(x_i) \right)}{\prod_{i=1}^{k} \Gamma(\alpha_X(x_i))} \prod_{i=1}^{k} p_X(x_i)^{\alpha(x_i)-1}$$

$$\sum p_X(x_i) = 1$$

$$\alpha_X(x_i) = r_X(x_i) + W \cdot a_X(x_i)$$

$$r_X(x_i) : \# \text{ observations of } x_i$$

$$a_X(x_i) : \text{ base rate of } x_i$$

$$E_X: \text{ Expected proba. distr.}$$

$$E_X = P_X$$

Example:

- 6 red balls
- 1 yellow ball
- 1 black ball
Multinomial Opinion $\leftrightarrow$ Dirichlet PDF

- **Dirichlet PDF evidence parameters:** $(r_X, a_X)$
- **Multinomial opinion parameters:** $(b_X, u_X, a_X)$

- $\text{Op} \rightarrow \text{Dir}$:
  
  \[
  \begin{aligned}
  r_X(x) &= \frac{W \cdot b_X(x)}{u_X} \\
  u_X + \sum b_X(x) &= 1 \\
  W &= 2
  \end{aligned}
  \]

- $\text{Dir} \rightarrow \text{Op}$:
  
  \[
  \begin{aligned}
  b_X(x) &= \frac{r_X(x)}{W + \sum r_X(x)} \\
  u_X &= \frac{W}{W + \sum r_X(x)}
  \end{aligned}
  \]
Non-informative prior weight: $W$

- Value normally set to $W = 2$.
- When $W$ is equal to the frame cardinality, then the prior Dirichlet PDF is a uniform.
- Normally required that the prior Beta is uniform, which dictates $W = 2$.
- Beta PDF is a binominal Dirichlet PDF.
- Setting $W > 2$ would make Dirichlet PDF insensitive to new observations, which would be an inadequate model.
Prior trinomial Dirichlet PDF, $W = 2$

Example:

Urn with balls of 3 different colours.
- $t_1$: Red
- $t_2$: Yellow
- $t_3$: Black

Ternary *a priori* probability density.
A posteriori probability density after picking:
- 6 red balls (t1)
- 1 yellow ball (t2)
- 1 black ball (t3)
- $W = 2$
Posterior trinomial Dirichlet PDF

A *posteriori* probability density after picking:

- 20 red balls (t1)
- 20 yellow balls (t2)
- 20 black balls (t3)

- $W = 2$
Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:
- 20 red balls (t1)
- 20 yellow balls (t2)
- 50 black balls (t3)
- $W = 2$
Hyper-Opinions

• Domain: \( X = \{x_1 \ldots x_k\} \)
• \( \mathcal{P}(X) \) is the powerset of \( X \)
• Hyperdomain \( \mathcal{R}(X) = \mathcal{P}(X) \setminus \{X, \emptyset\} \)
• \( \mathcal{R}(X) \) is the reduced powerset
• Hypervariable: \( X \in \mathcal{R}(X) \)
• Hyper opinion: \( \omega_X = (b_X, u_X, a_X) \)

• Belief mass distribution: \( b_X \) where \( b_X(x) \) is belief mass on \( x \in \mathcal{R}(X) \)
  \[ u_X + \sum_{X \in \mathcal{R}(X)} b_X(x) = 1 \]

• Base rate distribution: \( a_X \) where \( a_X(x) \) is base rate of \( x \in X \)
  \[ \sum_{X \in X} a_X(x) = 1 \]

• Proj. probability: \( P_X(x) = a_X(x) \cdot u_X + \sum_{x_j \in \mathcal{R}(X)} a_X(x \mid x_j) \cdot b_X(x_j) \)
Hyper Dirichlet PDF
Opinions v. Fuzzy membership functions

Fuzzy logic

Domain of fuzzy categories
- Tall
- Average
- Short

Crisp measures
- 250 cm
- 200 cm
- 150 cm
- 100 cm
- 50 cm
- 0 cm

Subjective logic

Domain of crisp categories
- Friendly aircraft
- Enemy aircraft
- Civilian aircraft

Uncertain measures
- \( \omega \)
Subjective Logic Operators
Homomorphic correspondence

- **Binary logic**
  - Booleans
  - Truth tables

- **Probabilistic logic**
  - Probabilities
  - PL operators

- **Subjective logic**
  - Opinions $\omega_X$
  - SL operators

Generalisation

- Homomorphic i.c.o. probability 0 or 1
- Homomorphic in case of absolute binomial opinions

Homomorphic correspondence

- $P(\omega_x \cdot \omega_y) = P(\omega_x) \cdot P(\omega_y)$ for probabilistic multiplication
- $B(\omega_x \cdot \omega_y) = B(\omega_x) \land B(\omega_y)$ for Boolean conjunction
### Subjective logic operators 1

<table>
<thead>
<tr>
<th>Opinion operator name</th>
<th>Opinion operator symbol</th>
<th>Logic operator symbol</th>
<th>Logic operator name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>+</td>
<td>∪</td>
<td>UNION</td>
</tr>
<tr>
<td>Subtraction</td>
<td>-</td>
<td>\</td>
<td>DIFFERENCE</td>
</tr>
<tr>
<td>Complement</td>
<td>¬</td>
<td>(\overline{x})</td>
<td>NOT</td>
</tr>
<tr>
<td>Projected probability</td>
<td>P(x)</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>Multiplication</td>
<td>·</td>
<td>∧</td>
<td>AND</td>
</tr>
<tr>
<td>Division</td>
<td>/</td>
<td>(\overline{\wedge})</td>
<td>UN-AND</td>
</tr>
<tr>
<td>Comultiplication</td>
<td>(\sqcup)</td>
<td>(\vee)</td>
<td>OR</td>
</tr>
<tr>
<td>Codivision</td>
<td>(\sqcap)</td>
<td>(\forall)</td>
<td>UN-OR</td>
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Subjective logic operators 2

<table>
<thead>
<tr>
<th>Opinion operator name</th>
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<th>Logic operator symbol</th>
<th>Logic operator name</th>
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<tbody>
<tr>
<td>Transitive discounting</td>
<td>⊗</td>
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<td>TRANSITIVITY</td>
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<td>Cumulative fusion</td>
<td>⊕</td>
<td>◊</td>
<td>n.a.</td>
</tr>
<tr>
<td>Averaging fusion</td>
<td>⊕</td>
<td>◊</td>
<td>n.a.</td>
</tr>
<tr>
<td>Constraint fusion</td>
<td>◦</td>
<td>&amp;</td>
<td>n.a.</td>
</tr>
<tr>
<td>Inversion, Bayes’ theorem</td>
<td>~</td>
<td>~</td>
<td>CONTRAPOSITION</td>
</tr>
<tr>
<td>Conditional deduction</td>
<td>◊</td>
<td>‖</td>
<td>DEDUCTION (Modus Ponens)</td>
</tr>
<tr>
<td>Conditional abduction</td>
<td>◊</td>
<td>‖</td>
<td>ABDUCTION (Modus Tollens)</td>
</tr>
</tbody>
</table>
Subjective Trust Networks
Trust transitivity

Thanks to Bob’s advice, Alice trusts Eric to be a good mechanic.

Bob has proven to Alice that he is knowledgeable in matters relating to car maintenance.

Eric has proven to Bob that he is a good mechanic.
Functional trust derivation requirement

- Functional trust derivation through transitive paths requires that the last trust edge represents functional trust (or an opinion) and that all previous trust edges represent referral trust.

- Functional trust can be an opinion about a variable.
Trust transitivity characteristics

Trust is diluted in a transitive chain.

Graph notation:
\[ [A, E] = [A; B] : [B; C] : [C, E] \]

SL notation:
\[ \omega_E^{(A;B;C)} = \omega_B^A \otimes \omega_C^B \otimes \omega_E^C \]

Computed with discounting/transitivity operator of SL
Trust Fusion
Combination of serial and parallel trust paths

Graph notation:

\[ [A, E] = \left( \left( [A;B] : [B;D] \right) \diamond \left( [A;C] : [C;D] \right) \right) : [D,E] \]

SL notation:

\[ \omega^E_{[A;B;D]} \diamond [A;C;D] = \left( \left( \omega^A_B \otimes \omega^B_D \right) \oplus \left( \omega^A_C \otimes \omega^C_D \right) \right) \otimes \omega^D_E \]
Discount and Fuse: Dilution and Confidence

Discounting dilutes trust confidence

Fusion strengthens trust confidence
Incorrect trust / belief derivation

Perceived: $([A, B] : [B, X]) \diamond ([A, C] : [C, X])$


Beware!
Hidden and perceived topologies

Perceived topology:

Hidden topology:

\[
([A, B] : [B, X]) \diamond ([A, C] : [C, X])
\neq ([A, B] : [B, D] : [D, X]) \diamond ([A, C] : [C, D] : [D, X])
\]

\((D, E)\) is taken into account twice
Correct trust / belief derivation

Perceived and real topologies are equal:

\[
( ([A; B] : [B; D]) \diamond ([A; C] : [C; D]) ) : [D, X]
\]
Computing discounted trust

\[ \omega^A_B : A \rightarrow B \]
\[ \omega^B_X : B \rightarrow X \]
\[ \omega^{(A;B)}_X : A \rightarrow X \]

A's trust in B

B's opinion about X

A's derived opinion about X

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Example: Weighing testimonies

- Computing beliefs about statements in court.
- $J$ is the judge.
- $W_1, W_2, W_3$ are witnesses providing testimonies.

\[ \omega_X^{(J;W_1)\Diamond(J;W_2)\Diamond(J;W_3)} \]
Simple Trust Network Demo

Four entities, labelled A, B, C and D have opinions about each other represented as points in triangles. Entity A is trying to form an opinion about D, and receives opinions from B and C as to the trustworthiness of D. Furthermore, A has his own opinions about the trustworthiness of B and C.

Left-click and drag opinion points to set opinion values. Entity A combines these opinions using the Subjective Logic Operators to derive his own opinion about D, as shown by the bottom opinion triangle. In detail, entity A discounts B's opinion about D by his opinion about B, and does similarly for C. Finally, he combines the two discounted opinions using the consensus operator in order to determine his opinion about D. Right-click on the opinion triangles to see the exact values of each opinion. Opinion values can also be visualised using three-coloured rectangles.

http://folk.uio.no/josang/sl/
Bayesian Reasoning
Deduction and Abduction

Parent node $X$  $\omega_X$

Causal conditionals

Child node $Y$  $\omega_{Y|X}$

Deduction

Abduction

$\omega_{X|Y}$
Deduction visualisation

- Evidence pyramid is mapped inside hypothesis pyramid as a function of the conditionals.
- Conclusion opinion is linearly mapped
Deduction – online operator demo

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Bayes’ Theorem

- Traditional statement of Bayes’ theorem:
  \[ p(x \mid y) = \frac{p(x)p(y \mid x)}{p(y)} \]

- Bayes’ theorem with base rates:
  \[ p(x \mid y) = \frac{a(x)p(y \mid x)}{a(y)} \]

- Marginal base rates:
  \[ a(y) = a(x)p(y \mid x) + a(\overline{x})p(y \mid \overline{x}) \]

- Bayes’ theorem with marginal base rates:
  \[ p(x \mid y) = \frac{a(x)p(y \mid x)}{a(x)p(y \mid x) + a(\overline{x})p(y \mid \overline{x})} \]
  \[ p(x \mid \overline{y}) = \frac{a(x)p(\overline{y} \mid x)}{a(x)p(\overline{y} \mid x) + a(\overline{x})p(\overline{y} \mid \overline{x})} \]
The Subjective Bayes’ Theorem

Binomial:

\[
(\omega_{x|y}, \omega_{x|\neg y}) = \tilde{\phi}(\omega_{y|x}, \omega_{y|\neg x}, a_x)
\]

Multinomial:

\[
\omega_{x|y} = \tilde{\phi}(\omega_{y|x}, a_x)
\]
Inversion Visualisation

(Subjective Bayes' theorem)
Deduction and abduction notation

\[ \omega_{Y \parallel Y} = \omega_X \otimes \omega_{Y \mid X} \]

\[ \omega_{X \parallel Y} = \omega_Y \odot \phi (\omega_{Y \mid X}, a_X) \]

\[ = \omega_Y \odot \omega_{X \mid Y} \]

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Example: Medical reasoning

- Medical test reliability determined by:
  - true positive rate \( p(y \mid x) \) where \( x \): infected
  - false positive rate \( p(y \mid \overline{x}) \) \( y \): positive test

- Bayes’ theorem:
  \[
p(x \mid y) = \frac{p(x)p(y \mid x)}{p(y)} = \frac{a(x)p(y \mid x)}{a(x)p(y \mid x) + a(\overline{x})p(y \mid \overline{x})}
  \]

- Probabilistic model hides uncertainty
- Use subjective Bayes’ theorem to determine \( \omega(\text{infected}) \)

  \[
  \omega_{X \mid Y} = \phi (\omega_{Y \mid X}, a_X)
  \]

- GP derives \( \omega(\text{infected} \mid \text{positive}) \) and \( \omega(\text{infected} \mid \text{negative}) \)
- Finally compute diagnosis \( \omega(\text{infected} \parallel \text{test result}) \)
- Medical reasoning with SL reflects uncertainty
Abduction – Online operator demo

http://folk.uio.no/josang/sl/
The General Idea of Subjective Networks

Bayesian Networks  Subjective Logic

Subjective Bayesian Networks  Subjective Trust Networks

Subjective Networks
Example SN Model

\[ \omega_Z^A = (((\omega_{B}^A \otimes \omega_{X}^B) \oplus (\omega_{C}^A \otimes \omega_{X}^C)) \odot \omega_{Y|X}^A) \odot \omega_{Z|Y}^A \]
Subjective Networks

Subjective Trust Network:

Subjective Bayesian Network:

Legend:  
- - - - Trust relationship
\[ \rightarrow \] Belief relationship
\[ \rightarrow \rightarrow \] Conditional relationship

Frame of variables

Frame of agents
New Book on Subjective Logic

A. Jøsang

Subjective Logic

A Formalism for Reasoning Under Uncertainty

Series: Artificial Intelligence: Foundations, Theory, and Algorithms

- A critical tool in understanding and incorporating uncertainty into decision-making
- First comprehensive treatment of subjective logic and its operations, by the researcher who developed the approach
- Helpful for researchers and practitioners who want to build artificial reasoning models and tools for solving real-world problems

This is the first comprehensive treatment of subjective logic and all its operations. The author developed the approach, and in this book he first explains subjective opinions,