A Appendix

A.1 Proof of Lemma 1

Proof. Fix a target \( n \), for any resource \( k \), we have:

\[
\begin{align*}
\bar{c}_n(D_0) &= E_{\bar{D}_0}[c_n(s)] = P_{\bar{D}_0}[c_n(s) = 1]
\end{align*}
\]

For the same resource \( k \) and any schedule \( l \), observe that \( \frac{\partial}{\partial \theta_{k,l}} \bar{c}_n(D_0) \) is therefore equal to:

\[
\begin{align*}
\frac{\partial}{\partial \theta_{k,l}} P_{\bar{D}_0}[c_n(s) = 1 | c_{-k,n}(s) = 0] P_{\bar{D}_0}(c_{-k,n}(s) = 0)
&= P_{\bar{D}_0}(c_{-k,n}(s) = 0) \frac{\partial}{\partial \theta_{k,l}} \sum_{d_k} c_{k,l,n} \theta_{k,l'}
&= (c_{k,l,n} - c_{k,0,n}) P_{\bar{D}_0}(c_{-k,n}(s) = 0).
\end{align*}
\]

A.2 Proof of Theorem 2

Proof. Given a parameterization \( \theta \), denote the probability that agent \( r_k \) is assigned to some schedule \( s_k = [t_{n_k,1}, \ldots, t_{n_k,L}] \) by \( p(s_k \mid \theta) \). Fixing a parameter \( w_{k,n} \), and subsequence of length \( l - 1 \), we can apply equation 6, to conclude that:

\[
\begin{align*}
\frac{\partial}{\partial w_{k,n}} P[s_k(l) = t_{n_k,l} \mid s_{k,1:(l-1)}, \theta]
&= \frac{\partial}{\partial w_{k,n}} \exp(w_{k,n}) \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n'})
&= 1[n = n_k, l] \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n'})
&= \exp(w_{k,n,l}) \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n,l'})
&= \frac{\left( \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n,l'}) \right)^2}{\exp(w_{k,n,l})}
&= 1[n = n_k, l] P[s_k(l) = t_{n_k,l} \mid s_{k,1:(l-1)}, \theta] \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n,l})
&= \exp(w_{k,n,l}) \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n,l})
&= 1[n = n_k, l] P[s_k(l) = t_{n_k,l} \mid s_{k,1:(l-1)}, \theta] \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n,l})
&= \sum_{t_{n,l} \in F(s_{k,1:(l-1)})} \exp(w_{k,n,l})
\end{align*}
\]

Equation 6 also tells us that:

\[
p(s_k \mid \theta) = \prod_{l=1}^{L} P[s_k(l) = t_{n_k,l} \mid s_{k,1:(l-1)}, \theta]
\]